Università degli Studi di Torino
CAUSTIC Journal Club

Will Multiple Probes of Dark Energy find Modified Gravity?

Shapiro, Dodelson, Hoyle, Samushia, Flaugher
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GR + DE
or
Modified Gravity

?

to analyze the data assuming that GR is correct and see whether the constraints on DE parameters overlap

→ no overlapping? → the underlying parameterization is wrong

to look at parameter constraints coming from separate dynamical effects (cosmic expansion, perturbation growth)

↓ Why the question?

GR is a bad fit to cosmological data unless a **new substance**, so called dark energy, is invoked

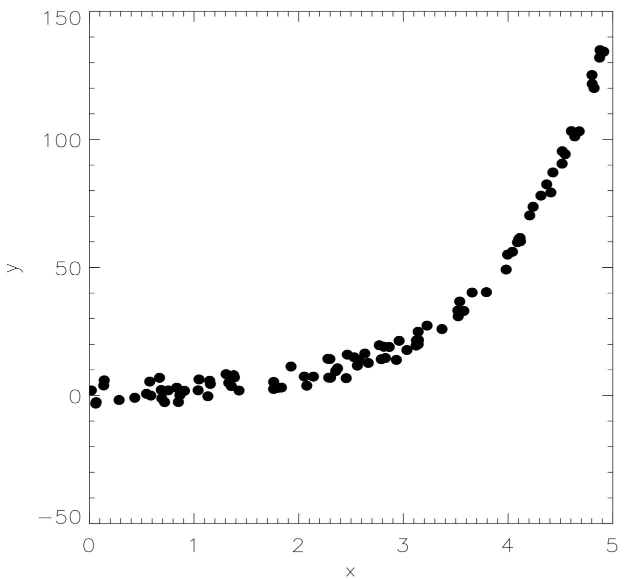
Concrete example

- The Universe is governed by a toy MG model
- Projected constraints from Dark Energy Survey (DES) in the plane (w_0,w_a), where $w = w_0 + w_a(1 - a)$

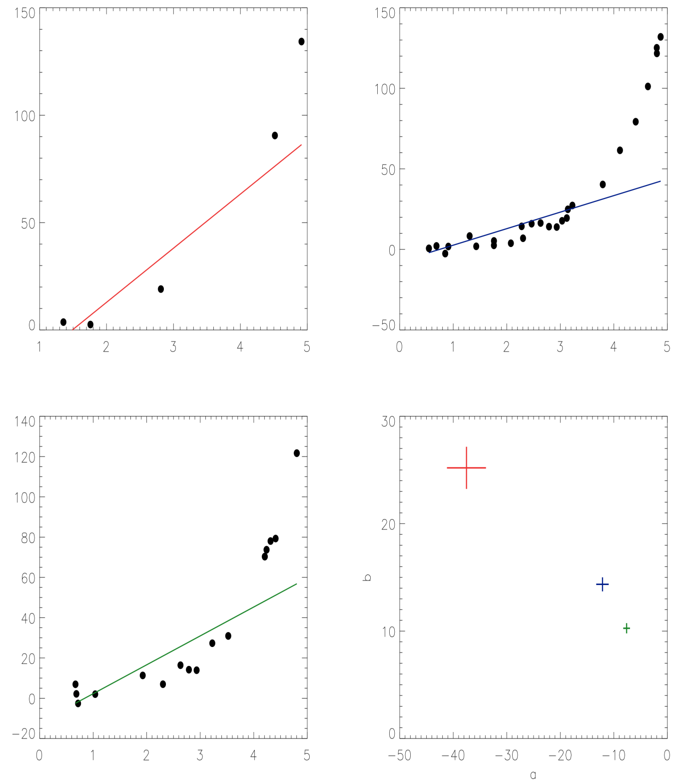
  a = scale factor
- Quantitative formalism that assigns a χ^2 for the combined probes

  => Bad $\chi^2 \rightarrow$ disagreement among the probes

Trivial Example



But we assume that a straight line is the correct model



★ Perturbations in MG: for the toy MG model, the metric retains its GR form

$$ds^2 = -(1+2\Psi)\,dt^2 + a^2(1+2\Phi)\,d\vec{x}^2$$

ψ,Φ = scalar gravitational potentials

★ Deviations from GR parameterized with (Hu & Sawicki, 2007)

$$g \equiv \frac{\Phi+\Psi}{\Phi-\Psi} \qquad f \equiv \frac{8\pi G\rho_m a^2\delta}{k^2(\Phi-\Psi)} - 1 \qquad \mu = \frac{1-g}{1+f}$$

GR: $g=0$ $f=0$ $\mu=1$

or (Linder 2005, Linder & Cahn 2007)

$$\frac{d\ln\delta}{d\ln a} = \Omega_m(a)^\gamma$$

Growth factor of matter perturbations

$\Omega_m(a) \equiv \Omega_{m,0}/[H(a)/H_0]^2$. $H(a)$ = expansion rate

In Hu & Sawicki formalism

$$\Omega_m(a)^{\gamma-1}\left[(1-2\gamma)\frac{d\ln H}{d\ln a} - 3\gamma + 2\right] + \Omega_m(a)^{2\gamma-1} = \frac{3}{2}\mu = \frac{3(1-g)}{2(1+f)}$$

GR: $\gamma=0.55$

Introduction
• Main question
• Example
• Cosmo context
• **Underlying model**
DES projections for *w*
MCT

Underlying TRUE model

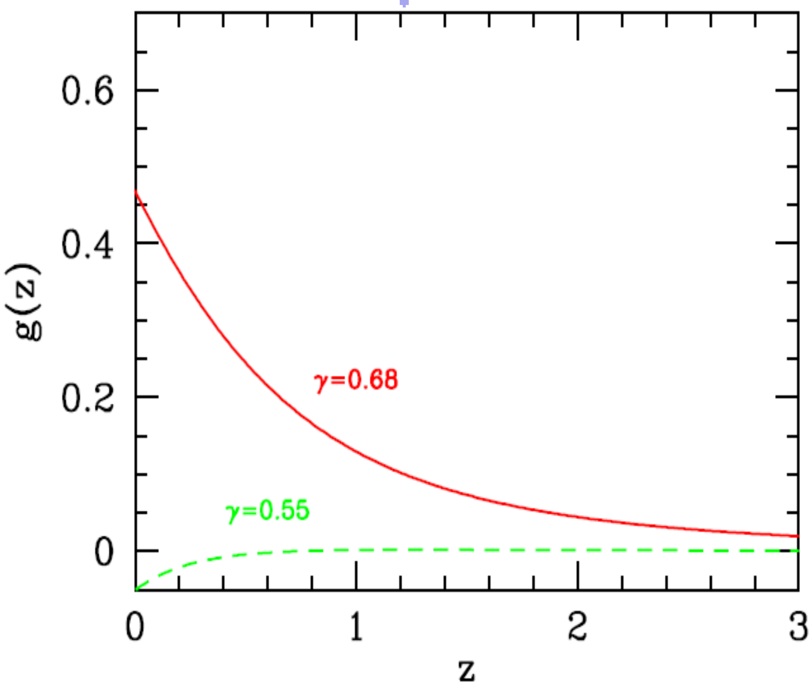
toy MG model

γ=0.68
f=0

} ~~**DGP**~~ w=1 → background expansion of ΛCDM

the only observable differences will enter via the growth function

mild modification of GR



⭐ Constant γ is not consistent with constant *g*

⭐ This model will produce more structure at early times than ΛCDM for fixed σ_8 (fluctuation amplitude today) because structure grows more slowly in the MG model

DES projections for *w*

will probe DE using

- Type Ia Supernovae (SN)
- High *z* Clusters (CL)
- Baryon Acoustic Oscillation (BAO) scales
- Cosmic Shear signal from weakly lensed galaxies (WL)

How to project these constraints?

FISHER MATRIX APPROACH
from contours representing the 68% confidence region in the (w_0,w_a) plane → **the assumed underlying model**

WE WANT TO

1- determine how large the error contours would be
2- determine where they would be centered

IF

an incorrect model is used to analyze the data

i.e. we use GR to fit the data but the toy MG model is the correct model

WARNING

The Fisher matrix formalism is valid when the joint likelihood function of the cosmological parameters is a GAUSSIAN.

Extension of Fisher formalism (Knox, Scoccimarro, Dodelson, 1998)

1- Calculate the Fisher matrix for the parameters λ_α to be fit to the data

$$F_{\alpha\beta} = \sum_{ij} (\mathrm{Cov}^{-1})_{ij} \boxed{\frac{\partial P_i}{\partial \lambda_\alpha} \frac{\partial P_j}{\partial \lambda_\beta}}$$

should be calculated using the model we will fit (GR)

P_i: observed quantity in bin i

$(\mathrm{Cov})_{ij}$: covariance matrix for bins i and j

should be calculated using the model assumed to be true (toy MG model)

Priors? They must be added to the Fisher matrix

2- Calculate the difference ΔP_i in the quantity to be measured P_i in the true model and in the fitted model

3- The parameter λ_α will be mis-estimated by an amount

$$\Delta\lambda_\alpha = \sum_\beta (F^{-1})_{\alpha\beta} \sum_{ij} (\mathrm{Cov}^{-1})_{ij} \frac{\partial P_i}{\partial \lambda_\beta} \Delta P_j$$

so we need to determine the expected values from the 4 probes in

- GR + DE
- MG model

we consider $\{w_0, w_a, \Omega_{DE}, \Omega_k, h, \Omega_b, n_s, \sigma_8\} = \{-1.0, 0.0, 0.73, 0, 0.72, 0.046, 1, 0.8\}$
+ priors from the Planck satellite.

Ω_k: curvature density | H(a)/(100 km/s/Mpc) | Ω_b: baryon density | n_s: slope of the primordial spectrum |
σ_8: normalizes the matter power spectrum at z=0

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DES projections for *w*
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  - **SN, BAO, CMB**
  - Clusters
  - Lensing
- Initial results

MCT

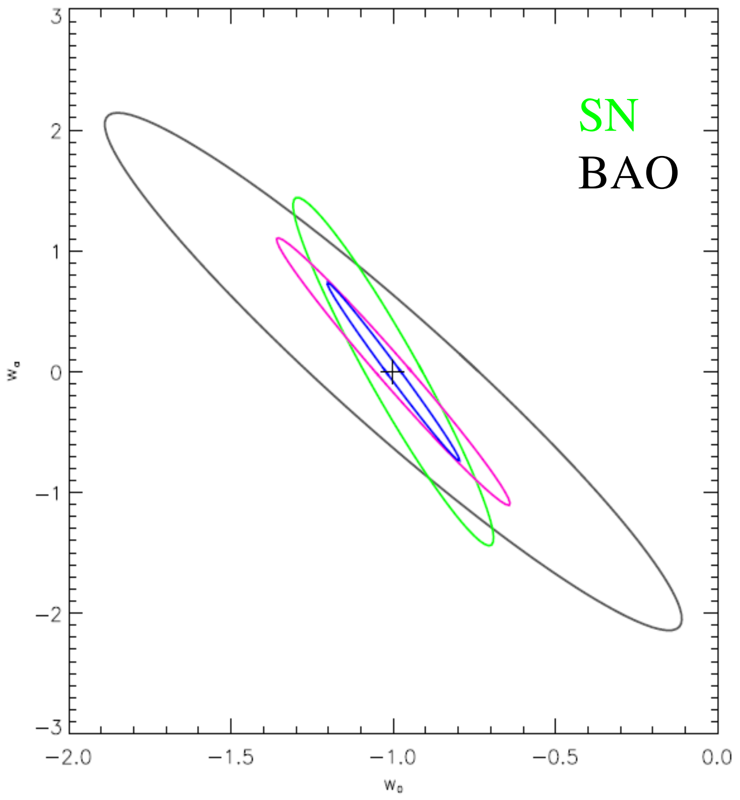
PROBES

SN, BAO, CMB

Sensitive to background geometry

The predictions for distance moduli and correlation function peak are identical for GR and MG

Insensitive because γ determines the structure growth in the late Universe (the CMB power spectrum is affected by ISW and WL but we ignore these effects)



$\Delta P = 0$ → **The projected contours are centered on the fiducial values**

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CLUSTERS

DES → optical
South Pole Telescope → microwaves (ZS effect)

Observable: number of clusters in each bin (of *z*) above a given mass threshold (which allows for detection by SPT)

Comoving number of clusters with mass *M* of *z*:

$$n(M,z) = -\frac{\rho_{c0}}{M}\frac{d\ln\sigma_M}{d\ln M}f(M,z)$$

$$f(M,z) = 0.316\exp\left(-1|\log\left(\sigma_M(z)^{-1}\right)+0.67|^{3.82}\right)$$ Jenkins et al. 2001

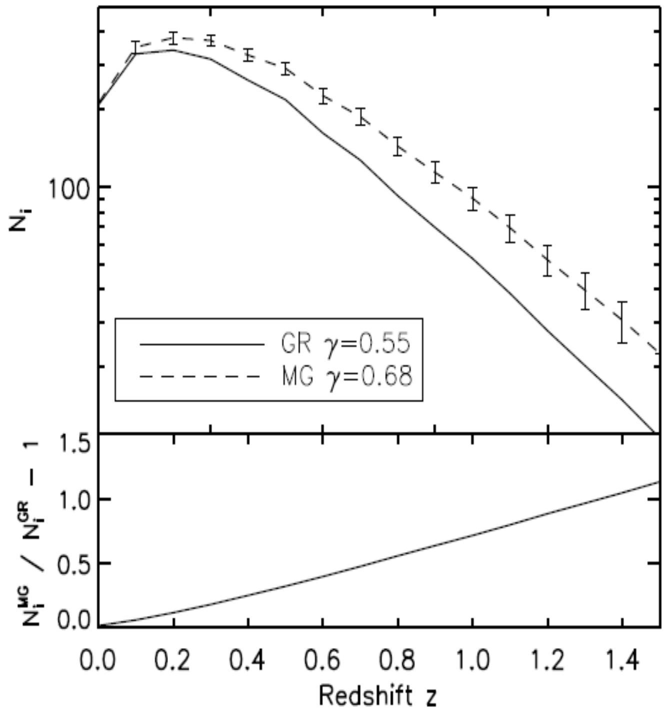
ρ_{c0} = critical density today

σ= RMS of the matter density field smoothed with a top-hat filter of radius R. $R^3 \equiv 3M/4\pi\rho_{c0}$.

Total number of clusters above $M_{\rm lim}(z)$:

$$N_i = 4\pi f^{\rm sky}\int_{z_i}^{z_{i+1}} dz\,\frac{\chi(z)^2}{H(z)}\int_{M_{\rm lim}(z)}^{\infty} dM\,n(M,z)$$

χ= comoving distance
z_i= lower edge of bin I
$f^{\rm sky}$= 0.125, sky coverage of DES+SPT



γ → linear growth function → it normalizes $P_{\rm lin}$ in σ

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A more general MG model would change — the halo formation time / the critical overdensity for halo collapse

but

DGP
f(R) gravities — alter δ_c by 1-2%

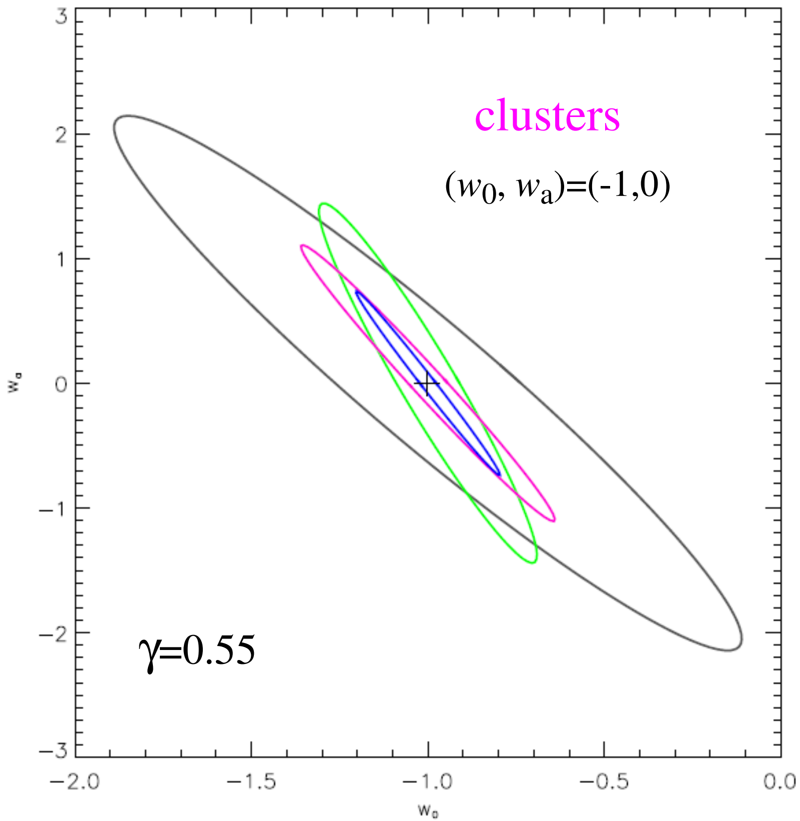
Changes in halo formation time are incorporated into the GR spherical collapse mass function (Sheth & Tormen, 2002)

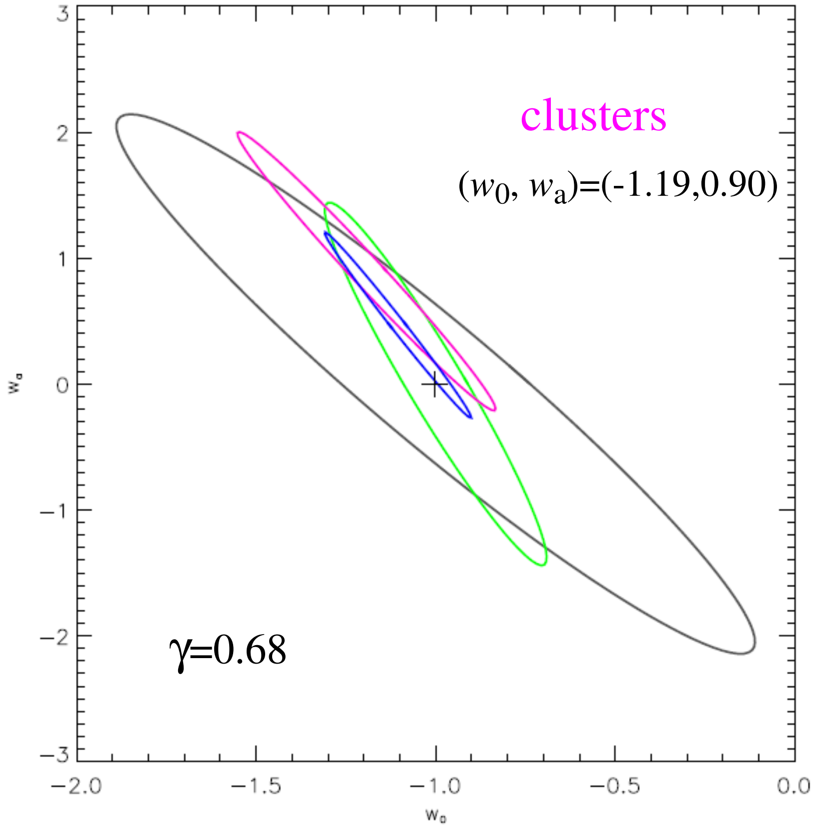
→ **cluster numbers depend primarily on the linear growth factor** — realistic

Covariance between bins: $\mathrm{Cov}[N_i, N_j] = \delta_{ij} N_i$ Assuming that the error in the number is dominated by counting error — WARNING

Projections in the dark energy parameter plane

To get the extra clusters that this model would produce, a larger *w* is needed (more DE at early times).





LENSING

★ The lensing convergence at a particular sky position is the matter density contrast $\delta(\boldsymbol{x})$, projected over comoving distance χ, along the line-of-sight:

$$\kappa_i(\theta) = \int_0^\infty d\chi\, \delta(\theta\chi, \chi)\, W_i(\chi)$$

i= redshift bin
$W_i(\chi)$= lensing kernel

★ Cosmic convergence power spectra: $\langle \tilde{\kappa}_i(\boldsymbol{l}) \tilde{\kappa}_j(\boldsymbol{l}') \rangle \equiv (2\pi)^2 \delta^2(\boldsymbol{l} + \boldsymbol{l}')\, C_{l;ij}$

small angles → no spherical harmonics

$C_{l;ij}$ = cross spectra
l= Fourier conjugate to θ
δ^2= 2-D Dirac function

★ For N redshift bins N(N+1)/2 observables for a given *l*:

$$C_{l;ij} = \int_0^\infty \frac{d\chi}{d_A(\chi)^2} W_i(\chi) W_j(\chi) P_\delta(k;\chi)$$

where d_A is modified in a curved Universe.

Limber approximation: the only matter density modes $\delta(\boldsymbol{x})$ contributing to the lensing signal are those modes with κ transverse to the l.o.s.

WARNING

★ Lensing kernel

$$W_i(\chi) = \frac{W_0}{n^{\rm gal}{}_i} \frac{d_A(\chi)}{a(\chi)} \int_\chi^\infty d\chi_s\, p_i(z) \frac{dz}{d\chi_s} \frac{d_A(\chi_s - \chi)}{d_A(\chi_s)} \qquad W_0 = \tfrac{3}{2}\Omega_m H_0^2$$

$p_i(z)$= true spectroscopic distribution of galaxies

$n^{\rm gal}$= total number density of galaxies in that bin

binning according to the "photo-z" of the galaxies, which is assumed as an unbiased estimator of the true redshifts

WARNING

★ Redshift distribution of source galaxies

$$\frac{dn^{\rm gal}}{dz} \propto z^2 \exp[-(z/z_0)^{1.5}]$$

Median redshift of distribution: $z_{\rm med} \approx z_0\sqrt{2}$,

★ Total projected number density of galaxies (normalized)

$$\int_0^\infty dz \frac{dn^{\rm gal}}{dz} = \sum_i n^{\rm gal}{}_i \equiv n^{\rm gal}$$

DES: $z_{\rm med}$=0.68, $n^{\rm gal}$=12

★ f= 0 → the relation between the lensing potential $\phi-\psi$ and the matter density δ is unchanged → the weak lensing power spectrum can be computed from the matter power spectrum via

$$C_{l;ij} = \int_0^\infty \frac{d\chi}{d_A(\chi)^2} W_i(\chi) W_j(\chi) P_\delta(k;\chi)$$

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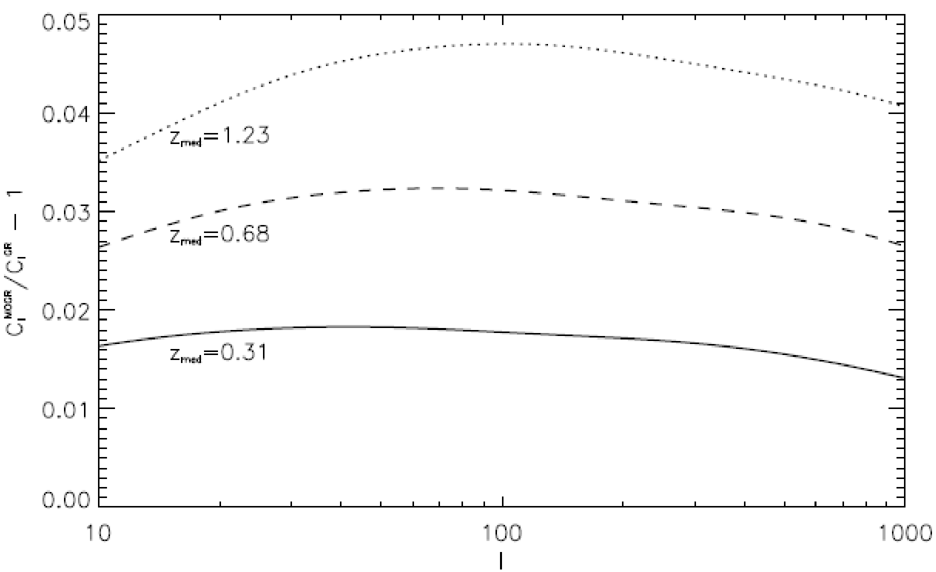
Non-linear power spectrum

	GR		MG
Linear	Fitting formula by Eisenstein & Hu (1999). Redshift dependence given by the growth function.	→	Growth function adapted to incorporate γ.
Non- linear	Halofit (Smith et al. 2003)		It must agree with GR at small non-linear scales (solar system). Halofit does not impose that.

Halofit (GR) —interpolation— Halofit (MG)



- ΔP_i: difference between the GR and the MG predictions for $C_{l;ii}$
- There is more power in the MG model at early times
- Cutoff at l=1000 → smaller scales contain non-linear baryonic effects

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- The total observed power in a given redshift bin is a combination of signal and noise

$$C^{\mathrm{obs}}_{l;ij} \equiv C_{l;ij} + \delta_{ij}\frac{\gamma^2_{\mathrm{rms}}}{n^{\mathrm{gal}}_i}$$

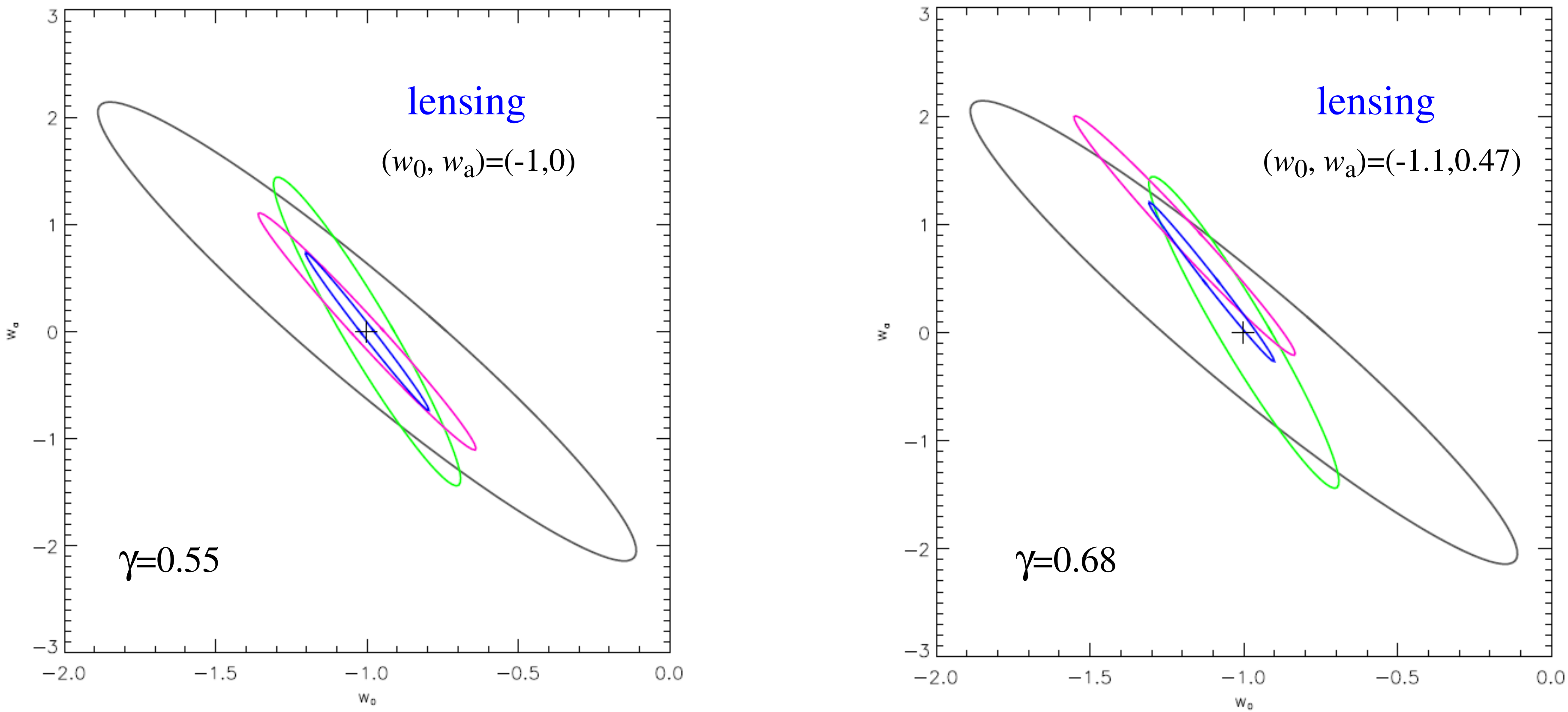
γ_{rms}=intrinsic scatter of one polarization of the galaxy shears

- Covariance between the observable spectra

$$\mathrm{Cov}[C^{\mathrm{obs}}_{l;ij}, C^{\mathrm{obs}}_{l';mn}] = \frac{\delta_{ll'}}{(2l+1)\Delta l\, f^{\mathrm{sky}}}\left(C^{\mathrm{obs}}_{l;im}C^{\mathrm{obs}}_{l;jn} + C^{\mathrm{obs}}_{l;in}C^{\mathrm{obs}}_{l;jm}\right)$$

f^{sky}= 0.12

γ_{rms}= 0.16



If the true model were MG, the lensing constraint would shift to more DE to accommodate the slower growth of structure

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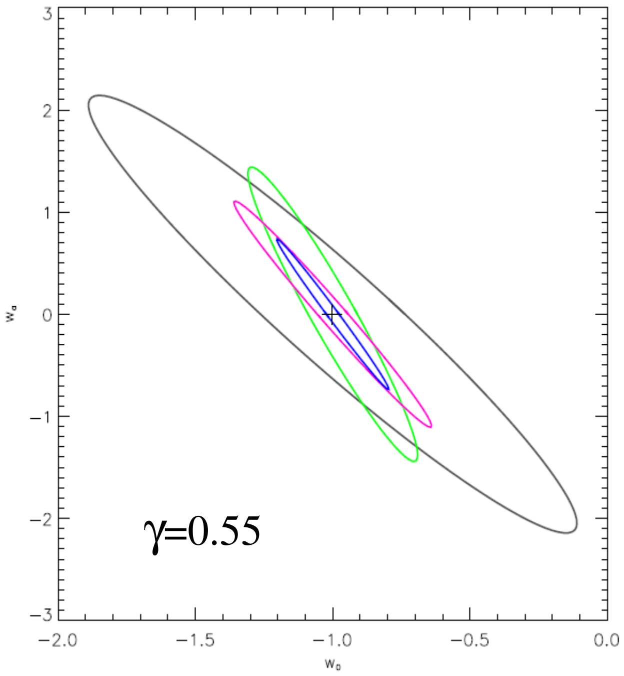
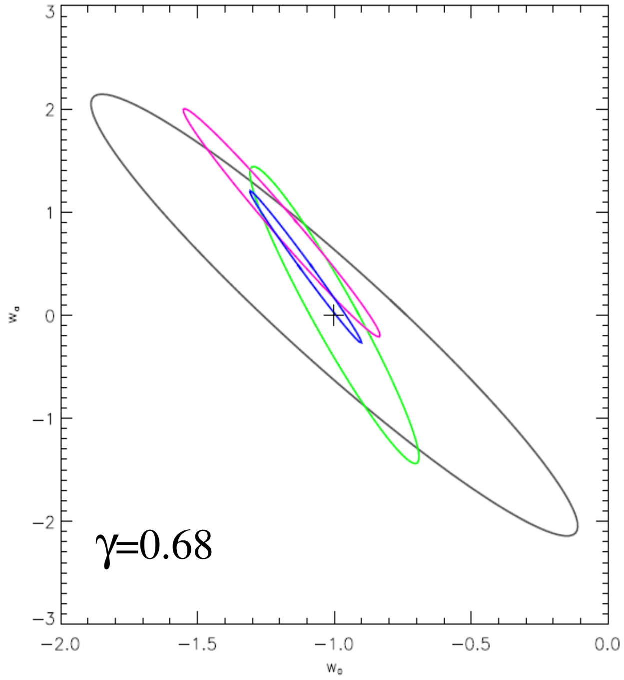
Initial results

from very different predictions



SAME RESULTS



the tension is not enough

WHY?

1- The approach is not quantitative ("do the contours overlap?")
→ no statistical conclusion

2- Each constraint is obtained with the Planck priors added in
→ the prior is used multiple times → redundant information!

3- The allowed regions are not 2-D but 8-D
→ it is possible that the allowed regions do not overlap in 8-D but do in the projections onto the 2-D subspace

NEW APPROACH

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DES projections for w
MCT
- Parameter tension
- Projections
- Parameter deg.
- Results

Multidimensional consistency test

Example: one free parameter l → M=1
2 probes → N=2

$$\chi^2(\lambda) = \sum_{i=1}^{2} \left(\lambda - \lambda^{(i)}\right) \frac{1}{[\sigma^{(i)}]^2} \left(\lambda - \lambda^{(i)}\right)$$

$\lambda^{(i)}$= best fit value of the parameter from the analysis of probe *i*
$\sigma^{(i)}$= error in probe *i*

✭ How to see if they are consistent?

- Minimize χ^2 with respect to λ
- $\lambda(\chi^2_{min})$ is the best fit value
- χ^2 quantifies the goodness of fit

Degrees of freedom: $\nu=(N-1)M=1$
Expectation value $<\chi^2_{min}>=\nu$

→ If $\chi^2_{min}=10 \rightarrow \Delta\chi=9$. For $\nu=1$ that means p=0.0026.

The probes are inconsistent with 99.7% confidence

✭ How to compute the tension between 2 probes if the assumed model is incorrect?

$<\lambda^{(1)}>=<\lambda^{(2)}>$ might be false → $<\chi^2_{min}>=\nu$ will not hold
$\therefore <\chi^2_{min}>=\nu+B$; B>0

If B=9 and $\nu=1$ we would conclude that the 2 probes are inconsistent with 99.7% confidence
→ B is the appropriate parameter to quantify the tension among several probes

8 cosmological parameters → M=8
5 probes (DES+Planck) → N=5

✯ Probe *i* returns a best fit set of parameters $\lambda_\alpha^{(i)}$ with a covariance matrix $C^{(i)}_{\alpha\beta}$ (invertible → no degeneracies)

✯ λ_α is a random point in cosmological parameter space

$$\chi^2(\lambda_\alpha) = \sum_i \sum_{\alpha\beta} (\lambda_\alpha - \lambda_\alpha{}^{(i)}) \left[C^{(i)}\right]^{-1}_{\alpha\beta} (\lambda_\beta - \lambda_\beta{}^{(i)})$$

quantifies the agreement of the probes

IF
• the likelihood from each probe is Gaussian in parameter space
• the assumed model is correct

WARNING

→ **excessively large values of χ^2 would falsify the underlying model**

how much? projections

• Compute the expectation value of χ^2 if the true model were MG
• Determine how much it exceeds (N-1)M=ν

$$\lambda_\alpha^{\min} = \sum_{\beta,\gamma} \left[\sum_j (C^{(j)})^{-1}\right]^{-1}_{\alpha\beta} \sum_i (C^{(i)})^{-1}{}_{\beta\gamma} \lambda_\gamma{}^{(i)}$$

$$\langle (C^{(i)})^{-1} \rangle = F^{(i)}$$

$$\left\langle \lambda_\alpha{}^{(i)} \lambda_\beta{}^{(j)} \right\rangle = \overline{\lambda}_\alpha{}^{(i)} \overline{\lambda}_\beta{}^{(j)} + \delta_{ij} (F^{(i)})^{-1}_{\alpha\beta}$$

$$\overline{\lambda}_\alpha{}^{(i)} \equiv \left\langle \lambda_\alpha{}^{(i)} \right\rangle$$

expected outcome from the *i*th experiment

$$\langle\chi^2_{\min}\rangle = (N-1)M + \sum_i \sum_{\alpha\beta} y_\alpha{}^{(i)} y_\beta{}^{(i)} (F^{(i)})^{-1}_{\alpha\beta} - \sum_{\alpha\beta} Y_\alpha Y_\beta (G^{-1})_{\alpha\beta}$$

where

$$y_\alpha{}^{(i)} \equiv \sum_\beta F_{\alpha\beta}{}^{(i)} \overline{\lambda}_\beta{}^{(i)} \qquad Y_\alpha \equiv \sum_i y_\alpha{}^{(i)} \qquad G_{\alpha\beta} \equiv \sum_i F_{\alpha\beta}{}^{(i)}$$

- If all probes are expected to return the same parameter → $\langle\chi^2_{\min}\rangle=(N-1)M$

the assumed model is correct

- Fiducial parameter set: $\Delta\lambda_\alpha{}^{(i)} \equiv \lambda_\alpha{}^{(i)} - \lambda_\alpha^{\mathrm{fid}}$

where $\Delta y_\alpha{}^{(i)} \equiv \sum_\beta F_{\alpha\beta}{}^{(i)} \Delta\overline{\lambda}_\beta{}^{(i)} \qquad \Delta Y_\alpha \equiv \sum_i \Delta y_\alpha{}^{(i)}$

$$\langle\chi^2_{\min}\rangle = (N-1)M + B$$

$$B \equiv \sum_i \sum_{\alpha\beta} \Delta y_\alpha{}^{(i)} \Delta y_\beta{}^{(i)} (F^{(i)})^{-1}_{\alpha\beta} - \sum_{\alpha\beta} \Delta Y_\alpha \Delta Y_\beta (G^{-1})_{\alpha\beta}$$

The projection for the excess of χ^2 due to inconsistency in the probes has been reduced to the calculation of B

$$\langle \chi^2_{\min} \rangle = (N-1)M + B$$

B=0 when there is no tension in the probes
- we fit the correct model
- the model we fit does not produce tension

N=5, M=8 → ν=32
If B=14.2 → $<\chi^2_{\min}>=\nu+B=46.2$

p=0.046 → the probability of finding a worse $\chi^2_{\min}$ than its expected value: $P(\chi^2_{\min}><\chi^2_{\min}>;\nu)$

The constraints from the 5 probes will be inconsistent at the 95.4 %

Parameter degeneracies

Experiment's insensitivity to a parameter (or combination) → Fisher matrix can be singular (non-invertible)

SN and BAO ↔ Non-geometric parameters: Ω_b, n_s, σ_8

CMB ↔ Cannot jointly constrain w_0,w_a,Ω_{DE}, Ω_k

} rows/columns=0

Degeneracies: error ellipsoids which are infinite in some directions in parameter space
→ no constraints in those directions → no inconsistency

Ex: σ_8 from SN cannot be inconsistent with σ_8 from CL, then σ_8 is not a degree of freedom in th SN error ellipsoid

Cleaning the Fisher matrix

• Find a unitary matrix U such that $F = U^T \Lambda U$

Λ : diagonal matrix of eigenvalues of F

• Replace the smallest element of Λ with zeros

• Compute F from Λ

this procedure negligibly changes the element of F provided that we only remove eigenvalues << the largest eigenvalue

• Compute $F^{-1} = U^T \Lambda^{-1} U$, where $(\Lambda^{-1})_{\alpha\alpha} = \begin{cases} 1/\Lambda_{\alpha\alpha} & \text{for } \Lambda_{\alpha\alpha} \neq 0 \\ 0 & \text{for } \Lambda_{\alpha\alpha} = 0 \end{cases}$

What happens to $\langle \chi^2_{min} \rangle$?

$$\langle \chi^2_{\rm min} \rangle = (N-1)M - \sum_i S^{(i)} + B$$

$S^{(i)}$: number of eigenvalues of $F^{(i)}$ which are zero

- Effectively, the number of degrees of freedom n has been reduced by the total number of parameters that each probe cannot constrain
- When very small eigenvalues are set to zero, B changes negligibly → no significant tension is expected among the probes in these highly degenerate directions

  → we evaluate tension only among the parameters where we expect tension
- Serious degeneracies: CMB, WL=3; SN, BAO, CL=4. ν=5x8-8=32

$\Sigma S^{(i)}$=4x3+3x2=18

=> ν_{eff}=14

Results

★ Suppose that we will mistakenly fit an 8-parameter ΛCDM model to data in a Universe described by the toy MG model

Scale-independent linear growth history given by γ=0.68

★ The expected tension among the DES probes and Planck (assuming only statistical errors) is

WARNING

WL	CL	SN	BAO	CMB	ν	B	$P(\chi^2_{\rm min} > \nu + B; \nu)$
	√		√	√	5	2.06	0.2164
	√	√		√	5	1.67	0.2466
	√	√	√		4	0.02	0.4030
	√	√	√	√	9	3.02	0.2121
√			√	√	6	2.08	0.2326
√		√		√	6	1.96	0.2414
√		√	√		5	0.75	0.3313
√		√	√	√	10	2.21	0.2715
√	√			√	6	6.71	0.0478
√	√		√		5	0.61	0.3462
√	√		√	√	10	8.23	0.0512
√	√	√			5	2.29	0.2003
√	√	√		√	10	9.22	0.0376
√	√	√	√		9	2.76	0.2271
√	√	√	√	√	14	15.58	0.0087

√: possible combinations

B: expected tension parameter

ν: effective degrees of freedom: MN-N-degeneracies

$<\chi^2_{\rm min}>=\nu+B$

If B is large => the constraints are inconsistent (non-overlapping)

P: goodness-of-fit or the probability of finding a worse $\chi^2_{\rm min}$ than its expected value (in a GR Universe)

→ probability that the probes would yield constraints with more tension than the tension predicted due to fitting an incorrect model

WL	CL	SN	BAO	CMB	ν	B	$P(\chi^2_{\rm min} > \nu + B; \nu)$
	√		√	√	5	2.06	0.2164
	√	√		√	5	1.67	0.2466
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√	√		√	√	10	8.23	0.0512
√	√	√			5	2.29	0.2003
√	√	√		√	10	9.22	0.0376
√	√	√	√		9	2.76	0.2271
√	√	√	√	√	14	15.58	0.0087

If either WL or CL is excluded → no significant tension among the probes → they are important

Only WL, CL and CMB: the probes disagree at 95%

When all DES probes are combined with Planck, the overlap in 8-D parameter space is very poor
→ they are inconsistent at 100(1-0.0087)=99.1%

Remarks

★ MCT is an improvement over the method of looking for overlap in (w_0, w_a)

→ It computes tension among ALL parameters

→ It does not use Planck priors multiple times

★ Only statistical errors. No systematic errors → they could degrade the parameter constraint

→ If the inconsistency seems large, look for systematics

★ Modest toy MG model which differs from ΛCDM only in the linear growth perturbations

→ Other MG models could easily produce more tension

★ Using the Multi-dimensional Consistency Test (MCT) future probes from DES will be able to rule out standard GR+DE IF the true gravity model is a modest modification of GR

The End