Future evolution of bound supercluster in an accelerating Universe

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Outline:

1. Theoretical framework
   - Cosmological model
   - A supercluster definition

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   - Main features
   - Identification of bound structures
   - Outcames

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3. A deeper analysis
   - Correlation function
   - The mass function
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Outline:

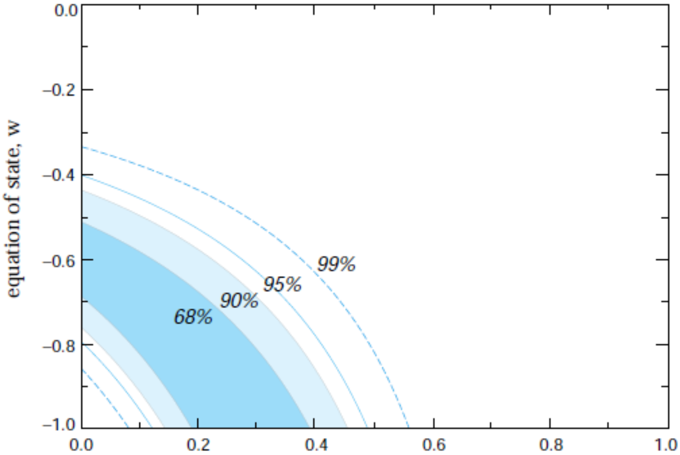
Cosmological model: ΛCDM flat Universe

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda) = \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

- $\Omega_m = 0.3$
- $\Omega_\Lambda = 0.7$
- $H_0 = 100\, h\, km\, s^{-1}\,\mathrm{Mpc}^{-1} = 70\, km\, s^{-1}\,\mathrm{Mpc}^{-1}$

Values chosen reffering to Chiueh & He (2002)

...obviously these values aren't the 'definitive' ones!



Five different time are studied:

$a = 1$ (today)

$a = 2$ (about 11 Gyr in the future)

$a = 5$ (in 25 Gyr)

$a = 10$ (in 37 Gyr)

$a = 100$ (in 78 Gyr)

A supercluster definition

In order to detect superclusters in the simulation, a criterion based on the spherical infall model is introducted.

The starting point of its derivation is the energy equation (per unit mass) of a shell:

$$E = \frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 - \frac{GM}{r} - \frac{\Lambda r^2}{6} = \mathrm{const.}$$

That can be written by introducing dimensionless variables:

$$\tilde{r} = \left(\frac{\Lambda}{3GM}\right)^{\frac{1}{3}} r$$

$$\tilde{t} = \left(\frac{\Lambda}{3}\right)^{\frac{1}{2}} t$$

$$\tilde{E} = \left(\frac{G^2M^2\Lambda}{3}\right)^{-\frac{1}{3}} E$$

$$\boxed{\tilde{E} = \frac{1}{2}\left(\frac{\mathrm{d}\tilde{r}}{\mathrm{d}\tilde{t}}\right)^2 - \frac{1}{\tilde{r}} - \frac{\tilde{r}^2}{2}}$$

Araya-Melo et al. define the **critical shell** as the shell

> *which separtes the region that will expand forever and the ones that will [...] turn around and fall in on to the core.*

That means the critical shell delimits the region of gravitational attraction from the repulsive one.

Or, with other words, the shell whose (dimensionless) radius maximizes the potential energy:

$$\tilde{V} = -\frac{1}{\tilde{r}} - \frac{\tilde{r}^2}{2}$$

$\tilde{V}$ is maximized for $\tilde{r} = 1$.
In this case, and assuming $\tilde{E}_{kin} = 0$, the energy for a shell will be $\tilde{E}_{cs} = -\frac{3}{2}$.

Substituting in the energy equation,

$$\tilde{E}_{cs} = -\frac{3}{2} = \frac{1}{2}\left(\frac{\mathrm{d}\tilde{r}_{cs}}{\mathrm{d}\tilde{t}}\right)^2 - \frac{1}{\tilde{r}_{cs}} - \frac{\tilde{r}_{cs}^2}{2} \, ,$$

one can integrate and find the evolution of the critical shell's radius.

Once defined $\tilde{r}_{cs}$, is straightforward to write a **density condition for bound shells**:

$$\frac{\rho_{shell}}{\rho_{cr}} \geq \frac{\rho_{cs}}{\rho_{cr}} = \frac{2\Omega_\Lambda}{\tilde{r}_{cs}^3} = 2.36$$

$$1+\delta = \frac{2\Omega_{\Lambda 0}}{\Omega_{m0}} \left(\frac{a}{\tilde{r}}\right)^3$$

Dünner et al. (2006) used N-body simulation for testing this results:
only 0.3 % of the mass really bound to an object is not enclosed within r_{cs}, but inside the critical shell there is also a large amount of unbound matter ($\sim 30\%$).

Computer simulation

Main features:

- GADGET-2 code (Springel 2005)
- $500\, h^{-1}\mathrm{Mpc}$ side length box
- 512^3 dark matter particles
- $m_{DM} = 7.75 \times 10^{10} h^{-1} M_{\odot}$
- I.C. at redshift $z = 49$ $(a = 0.02)$

  [no more details provided]
- Five snapshot at $a = 1, 2, 5, 10, 100$

Identification of bound structures

1st step: fix a lower limit in object definition (completeness of the sample)

$$N_{obj} \geq 50 \text{ particles} \quad \Leftrightarrow \quad M \geq 3.1 \times 10^{13} h^{-1} M_{\odot}$$

2nd step: use the HOP algorithm (Eisenstein & Hut 1998) to find groups of particle (i.e. theDM halos)

3rd step: for each group, consider the virialized core and calculate the centre of mass

4th step: apply the **critical shell criterion** in order to identify the bound region around the evaluated centre.

Identifiation of structures: supercluster

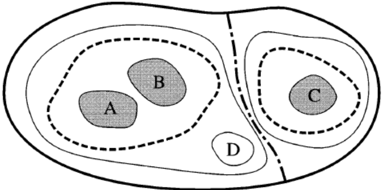
When two different critical shell (i.e. bound region) overlap each other, they are considered as parts of the same structure.

A *supercluster* is such a structure with $M \geq 10^{15} h^{-1} M_{\odot}$.

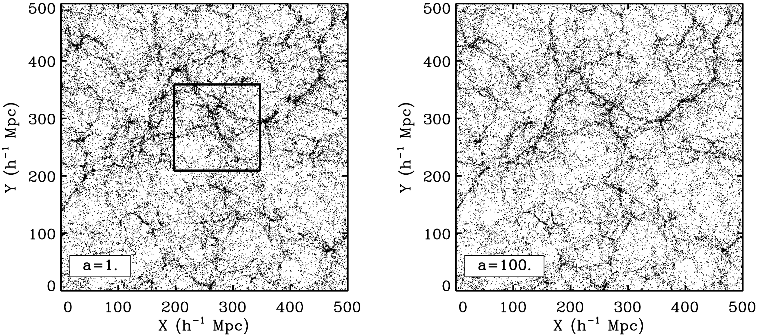
Less massive merged object are simply called, by Araya-Melo et al., *bound objects*.

A glimpse of HOP

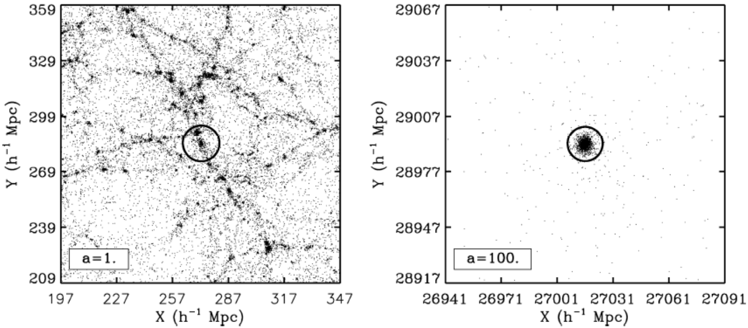
- Assign to each particle an estimate of its local density.
- Associate every particle to the densest one among the N_{HOP} nearest.
- Reach the particle that is it own densest neighbor (δ_{peak}).
- All the particle on this path are in the same group.
- Combine group that share some density contour (δ_{saddle})



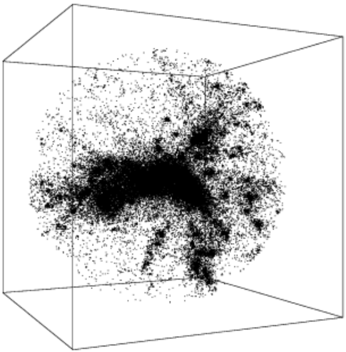
Outcomes of the simulation



While in comoving coordinates the accelerated expansion [...] results in a freezing of structure growth on scale much larger than the initial size of superclusters...

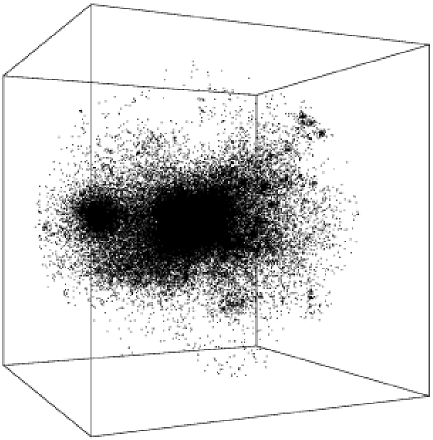


...in physical coordinates the separation of structures continues and grows exponentially in time.

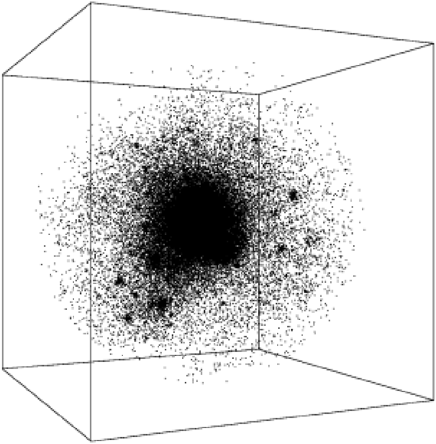


One of the most massive objects in the simulation:
$M \sim 6.8 \times 10^{15} h^{-1} M_{\odot}$ at $a = 1$.
Physical size of the box: $14 h^{-1}\mathrm{Mpc}$ at every a.

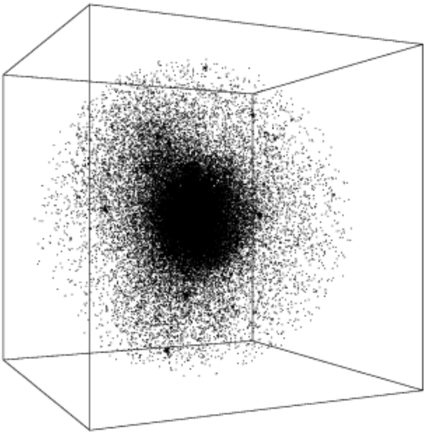
$a = 2$



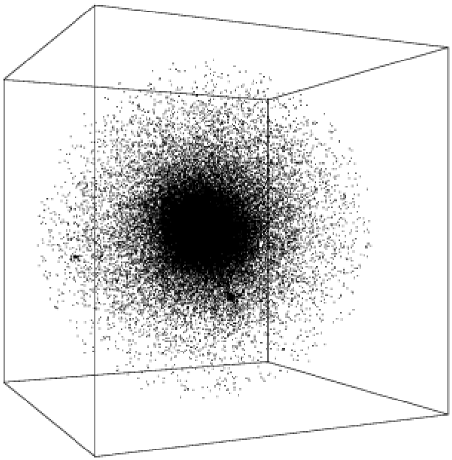
$a = 5$



a = 10



$a = 100$



Some number...

- At $a = 1$ HOP finds $\sim$ 20600 'clusters' (i.e. virialized halos).
- At $a = 100$ the clusters are decreased to $\sim$ 18000.
- Bound object (i.e. overlapping clusters) are $\sim$ 4900 at every age.
- Among these, $\sim$ 535 are supercluster,
- 17 of them very massive $(M > 5 \times 10^{15} h^{-1} M_{\odot})$
- $\sim$ 70% of the mass remain outside of the superclusters

A deeper analysis: correlation function $\xi(r)$

Cluster and supercluster correlation functions do not significantly change over this time interval, and retain, approximately, their power-law behavior $\left(\frac{r}{r_0}\right)^{-\gamma}$.

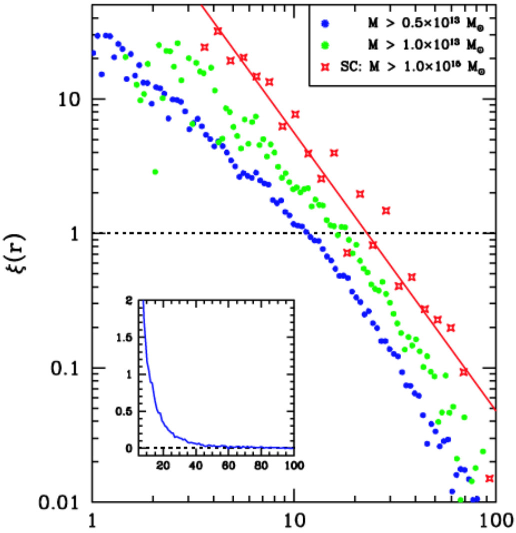
For superclusters at $a = 100$:

$$r_0 = 23 \pm 5 h^{-1}\mathrm{Mpc}$$

$$\gamma \simeq 2.1$$

$\xi(r)$ 0 within $r \gtrsim 100\, h^{-1}\mathrm{Mpc}$ *

(*) This clustering scaling mass has to some extent to be related to the properties of the peaks in the Gaussian primordial density field. *[!?]*


M > 0.5×10¹³ M⊙
M > 1.0×10¹³ M⊙
SC: M > 1.0×10¹⁵ M⊙
ξ(r)
10
1
0.1
0.01
1
10
100
2
1.5
1
0.5
0
20
40
60
80
100

The mass function

A comparison between simulation, PS and ST.

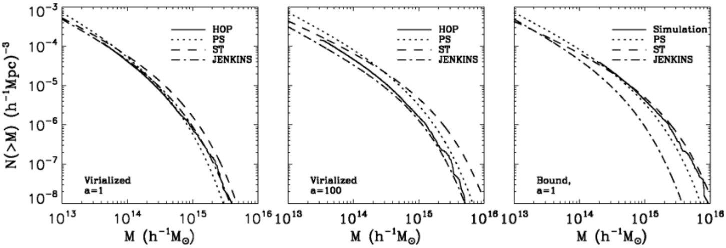


Figure 9. Mass functions of virialized haloes and of bound objects compared with three theoretical mass functions. These are the Press–Schechter mass function (dotted line), the Sheth–Tormen function (dashed line) and the Jenkins function (dot–dashed line). For the virialized HOP haloes, the critical overdensity δ_c for the PS and ST functions is the one for collapse, for the bound objects the value δ_b for assuring a bound object (Paper I; this study). Left-hand panel: the $a = 1$ integrated mass function $N(>M)$ of HOP haloes. Centre: the $a = 100$ integrated mass function $N(>M)$ of HOP haloes. Right-hand panel: the $a = 100$ integrated mass function $N(>M)$ of bound objects.

The authors include also the heuristic function suggested by Jenkins et al. (2001).

Shapes of the clumps

Inertia tensor

$$I_{ij} = \sum x_i x_j m$$

with eigenvalues $(\lambda_1, \lambda_2, \lambda_3)$.

$$\frac{s_2}{s_1} = \sqrt{\frac{\lambda_2}{\lambda_1}}, \qquad \frac{s_3}{s_1} = \sqrt{\frac{\lambda_3}{\lambda_1}}$$

A distribution is spherical if

$$\frac{s_2}{s_1} \simeq \frac{s_3}{s_1} \simeq 1$$

Sometimes the reader has the impression that this paper is quite too... sensationalist...

> *The fact that are almost no spherical, or even nearly spherical objects, at the earlier epochs is hardly surprising [...]*

Is it?

Primordial spectrum, Zeldovich approsimation, ...

Multiplicity of a supercluster

Mean mass of clusters $9.4 \times 10^{13}\, h^{-1} M_{\odot}$
Mean mass of cluters within Scl. $3.6 \times 10^{14}\, h^{-1} M_{\odot}$

$\Rightarrow$ Clusterization process is stronger at higher masses.
This is quite intresting: see Kaiser (1984)

In this simulation all detected superclusters have less than 15 members. Already at a $= 2$, the average number of clusters within the same bound structure is ~ 3.

Summarizing

- A dark matter box is made to evolve to a $= 100$
- Given a definition of *critical shell*, and by mean of HOP algorithm, bound sturctures are indetified
- Opposite to a radical internal evolution, on larger scale the scenario is virtually frozen
- Some not-so-impressive analysis is carried on.

There is a Shapley Supercluster?

In Araya-Melo et al. simulation, the biggest supercluster has a mass of about $M \sim 7 \times 10^{15}\, h^{-1} M_{\odot}$.

By mean of their mass function, it's possible to estimate that in the range $0 < z < 0.1$ the most massive structure may have $M_{SC} = 8 \times 10^{15}\, h^{-1} M_{\odot}$

Dünner et al. (in prep.) claim the mass of Shapley SCl. is nearly $7 \times 10^{15}\, h^{-1} M_{\odot}$,
calculated using some tools of Araya-Melo et al.

There is a Shapley Supercluster?

Clearly, there is no agreement with respect to multiplicity:

$$N_{Araya} \leq 15 \text{ members}$$

$$N_{SSC} \sim 40 \text{ memb.} \qquad \text{(according to Proust 2006)}$$

The *Caustic Technique* evaluation yelds to $M_{SSC} \simeq 5 \times 10^{15} h^{-1} M_{\odot}$.
Otherwise, Muñoz & Loeb (2008) find $M_{SSC} \sim 4 \times 10^{16}$

Question?

- What does this work really add to a simple (but effective) analytical study?
- Wouldn't be better to set many simulations with different cosmologies?
- Will the human race live enough to check if this simulation was right?
- ...

Spatial distribution of bound object

filling factor

$$f = \sum_i \frac{V_i}{V_{box}}$$

For all bound structures at a=1

$$3.34 \times 10^{-2}$$

And at a=100

$$4.14 \times 10^{-8}$$

of which 23% superclusters.

We propose a physically motivated definition of superclusters as the largest structures that will remain gravitationally bound as they separate at an exponentially increasing rate in the dark-energy-dominated future of the Universe. Using the spherical collapse model, we were able to analytically determine the condition by which a spherical shell will eventually stop its expansion, becoming the outer limit of a gravitationally bound structure. In particular for the present universe, this criterion states that only shells containing a mean density of 2.36 times the critical density (ρ_c), will eventually stop growing. We tested our criterion using N-body simulations, showing that it gives a good estimate of the external limit of bound structure, and accordingly overestimating its bound mass. The model also showed to give a good estimation of radial velocities up to deep inside the core of the structure. Using this information, we generated a method to estimate the critical shell of structures as seen in redshift space. This method relies in great amount on its calibration using numerical simulations. We applied our method