The Caustic Technique

An overview

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CAUSTIC Journal Club

About the mass of galaxy clusters
- Why is it relevant?
- **How to calculate it?**

Caustic Technique
- Theory
- Details
- Applications
- Problems
- Byproducts

Conclusions

Why the mass of galaxy clusters ?

Mass distribution on intermediate scales (1-10 Mpc/h)

Small scales → assumption of dynamical equilibrium

Large scales → small over-densities → linear theory

mass density fluctuations and peculiar velocities evolve under the effect of gravity

Exponential tail of mass function → probe of the cosmological parameters

80% - DM
20% - hot diffuse plasma
- stars, dust, cold gas (gxs)

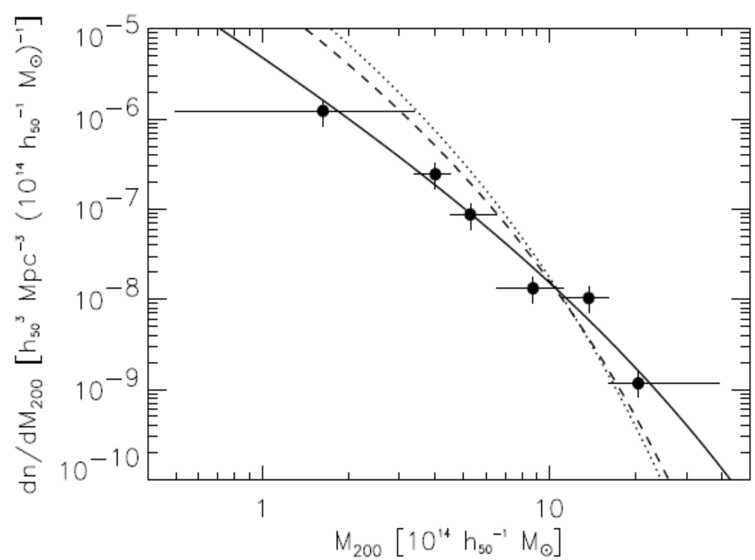

Fig. 4 Mass function of the HIFLUGCS X-ray clusters (dots with error bars). The solid line is the best fit with $\Omega_m = 0.12$ and $\sigma_8 = 0.98$. The dashed and dotted lines are the best fits with $\Omega_m = 0.5$ or $\Omega_m = 1.0$ held fixed, which yield $\sigma_8 = 0.60$ and $\sigma_8 = 0.46$, respectively. From Reiprich & Böhringer (2002).

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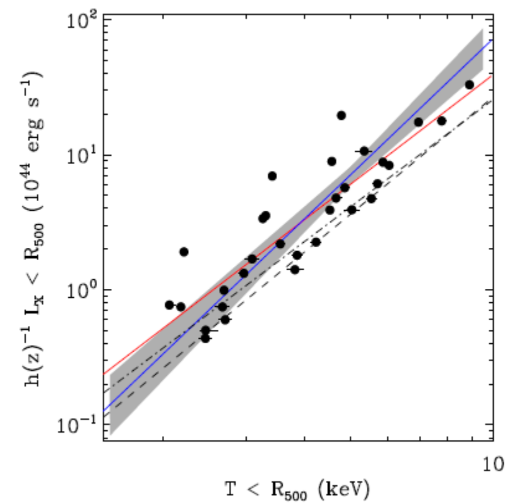
How the mass of galaxy clusters ?

Can we assume dynamical equilibrium?

From scaling relations... ~yes



Pratt et al. 2009

However there are...

spatially inhomogeneous thermal and non thermal emission

kinematic and morphological segregation of galaxies

signature of non-gravitational processes

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dynamical equilibrium + sphericity

total mass within a radius | mass profile

from l.o.s. velocities and position

virial theorem

$$M = \frac{3\sigma^2 R}{G}$$

$$R = \frac{\pi}{2}\frac{N(N-1)}{2}\left(\sum_i \sum_{i>j} \frac{1}{r_{ij}}\right)^{-1}$$

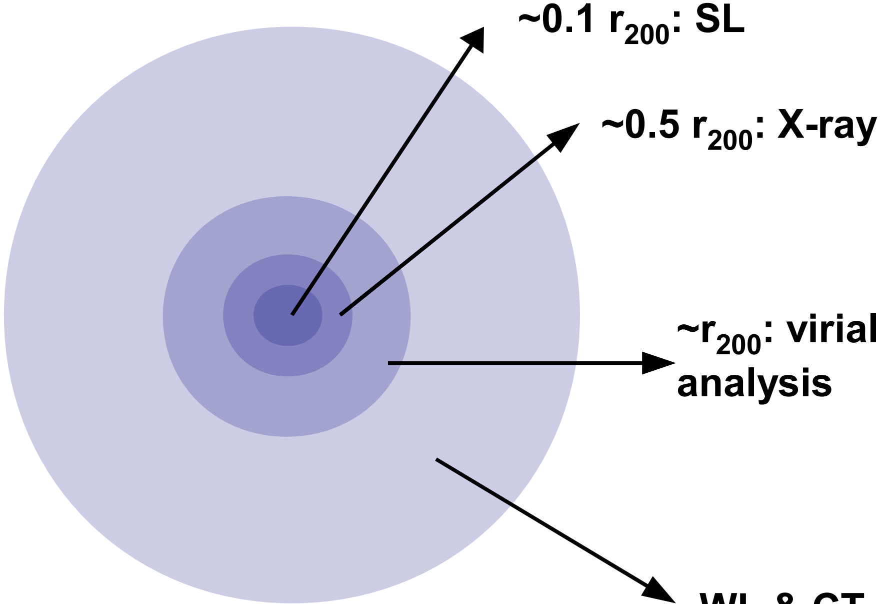
scaling relations

Jeans equation

$$M(<r) = -\frac{\langle v_r^2 \rangle r}{G}\left[\frac{\mathrm{d}\ln\rho_{\mathrm{m}}}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln\langle v_r^2\rangle}{\mathrm{d}\ln r} + 2\beta(r)\right]$$

$$\beta(r) = 1 - \frac{\langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle}{2\langle v_r^2 \rangle}$$

- Mass-anisotropy degeneracy
- Assumption of relation between the gx density profile and the mass density profile



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total mass within a radius | mass profile

from X-ray

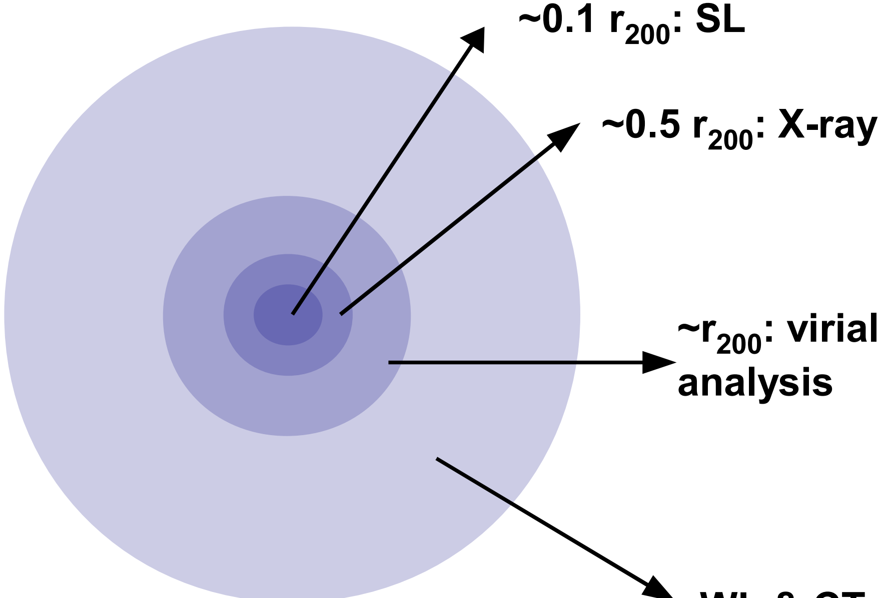
X-ray temperature
- Isothermal ICM → important departure from assumption in some clusters

scaling relations
- The complex thermal structure of the ICM can bias the estimate.

X-ray spectrum

$$M(<r) = -\frac{kTr}{G\mu m_{\rm p}}\left[\frac{{\rm d}\ln\rho_{\rm gas}}{{\rm d}\ln r} + \frac{{\rm d}\ln T}{{\rm d}\ln r}\right]$$

- No β → the ICM pressure is isotropic



~~dynamical equilibrium + sphericity~~

total mass within a radius — mass profile

from lensing

Strong lensing

- Multiple images, arcs

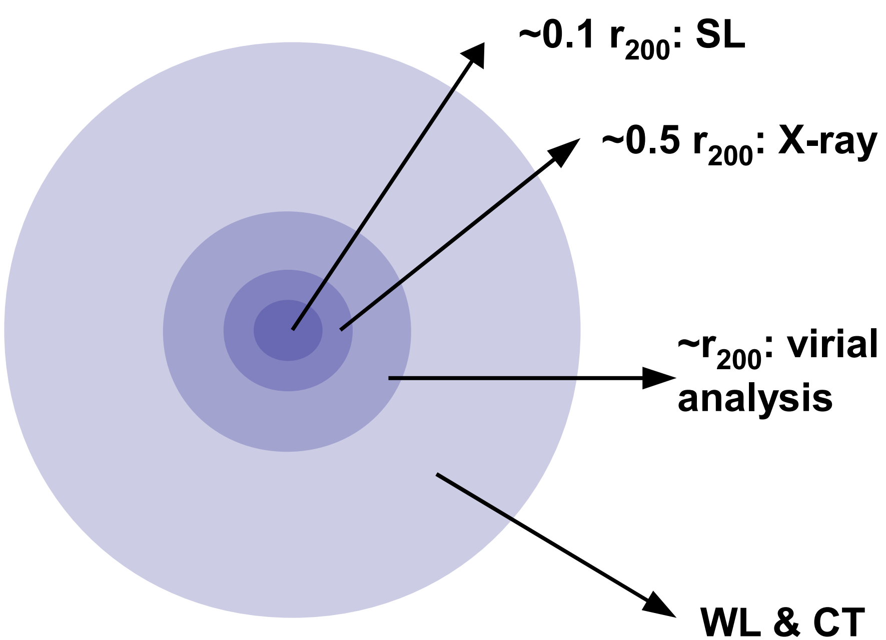
$$M(<r) = \frac{rc^2}{4G}\alpha$$

Weak lensing

- Tangential distortion of the shape of the background galaxies

Disadvantage: the signal intensity depends on the distances between observer , lens and source

from l.o.s. velocities and position



Caustic Technique

•In hierarchical models of structure formation, the velocity field surrounding the cluster is not perfectly radial, as expected in the spherical infall model

↓

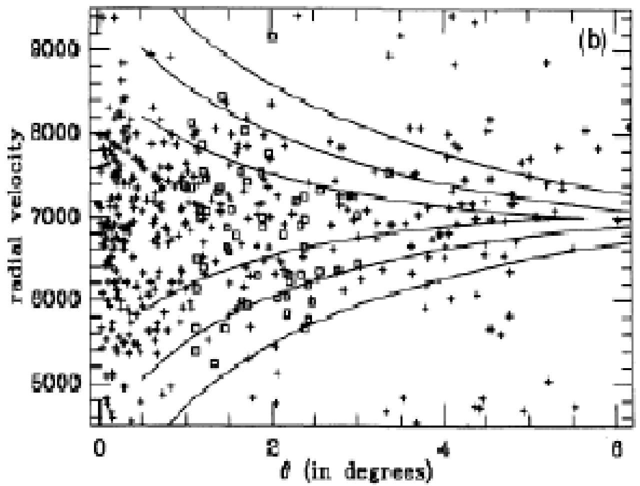
- It is possible to extract the escape velocity of galaxies from the redshift diagram

Caustic Technique - Theory

✯ When observed in redshift space, the infall pattern around a rich cluster appears as a “trumpet” whose amplitude $A(\theta)$ decreases with θ.

✯ Turn around radius → $A(\theta)=0$. The caustics $A(\theta)$ are located where the galaxy number density in redshift space is infinite.

$$\mathcal{A}(\theta) \sim \Omega_0^{0.6} r f(\delta) \left[-\frac{\mathrm{d}\ln f(\delta)}{\mathrm{d}\ln r} \right]^{-1/2}$$



Fuzzy caustics in dense sampling of the infall region of a rich cluster

✯ But clusters accrete mass ellipsoidally and anisotropically
→ the velocity field can have a substantial non-radial random component

$$A(\theta)_{\text{infall model}} < A(\theta)_{\text{non-radial random components}}$$

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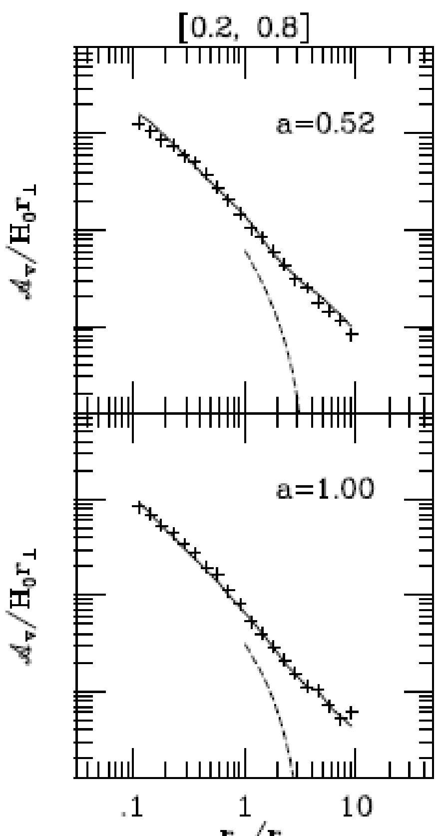
Interpretation

$A(\theta)$ is the average over a volume $d^3\boldsymbol{r}$ of the square of the l.o.s. component of the escape velocity

$$\langle v^2_{\rm esc,los}(r)\rangle = \left[-2\phi(r)g^{-1}(\beta)\right]^{1/2} \qquad g(\beta) = \frac{3-2\beta(r)}{1-\beta(r)}$$

$\mathcal{A}^2(r) = \langle v^2_{\rm esc,los}(r)\rangle$ **HOLDS INDEPENDENTLY OF THE DYNAMICAL STATE OF THE CLUSTER**



The caustic amplitude and the $<v^2_{\rm esc,los}(r)>$ agree at all scales out to 10 virial radii, independently of the dynamical state of the cluster

Effect of the cluster shape: to yield different amplitudes depending on the l.o.s.

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- From $A(\theta) = \langle v^2_{esc,los}(r) \rangle \rightarrow -2\phi(r) = \mathcal{A}^2(r)g(\beta) \equiv \phi_\beta(r)g(\beta)$
- Mass of an infinitesimal shell

$$G\,\mathrm{d}m \; = \; -2\phi(r)\mathcal{F}(r)\,\mathrm{d}r \; = \; \mathcal{A}^2(r)g(\beta)\mathcal{F}(r)\,\mathrm{d}r$$

where $\mathcal{F}(r) = -2\pi G \frac{\rho(r) r^2}{\phi(r)}$

therefore

$$GM(<r) = \int_0^r \mathcal{A}^2(r)\mathcal{F}_\beta(r)\,\mathrm{d}r$$

$$\mathcal{F}_\beta(r) = \mathcal{F}(r)g(\beta)$$

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☆ From $A(\theta) = \langle v^2_{esc,los}(r) \rangle$ → $-2\phi(r) = \mathcal{A}^2(r) g(\beta) \equiv \phi_\beta(r) g(\beta)$

☆ Mass of an infinitesimal shell

$$G\,\mathrm{d}m = -2\phi(r)\mathcal{F}(r)\,\mathrm{d}r = \mathcal{A}^2(r) g(\beta)\mathcal{F}(r)\,\mathrm{d}r$$

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therefore

$$GM(<r) = \int_0^r \mathcal{A}^2(r)\mathcal{F}_\beta(r)\,\mathrm{d}r$$

$$\mathcal{F}_\beta(r) = \mathcal{F}(r) g(\beta)$$

mass profile

☆ $\mathcal{F}_{\rm NFW}(r) = \frac{r^2}{2(r+r_s)^2}\frac{1}{\ln(1+r/r_s)}$ is a slowly changing function of r, and

also $\mathcal{F}_\beta(r)$ if $g(\beta)$ is

$$\mathcal{F}_\beta(r) = \mathcal{F}_\beta$$

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$$\mathcal{F}_\beta(r) = \mathcal{F}_\beta \quad \Rightarrow \quad GM(<r) = \mathcal{F}_\beta \int_0^r \mathcal{A}^2(r)\,\mathrm{d}r$$

originally F_β=0.5

HEURISTIC RECIPE FOR THE ESTIMATION OF THE MASS PROFILE

Details – Major steps

- arrange the galaxies in a binary tree according to a hierarchical method
- select two thresholds to cut the tree: the largest group obtained from the upper-level threshold identifies the cluster candidate members
- build the redshift diagram of all the galaxies in the field; locate the caustics, and identify the final cluster members
- the caustic amplitude determines the escape velocity and mass profiles.

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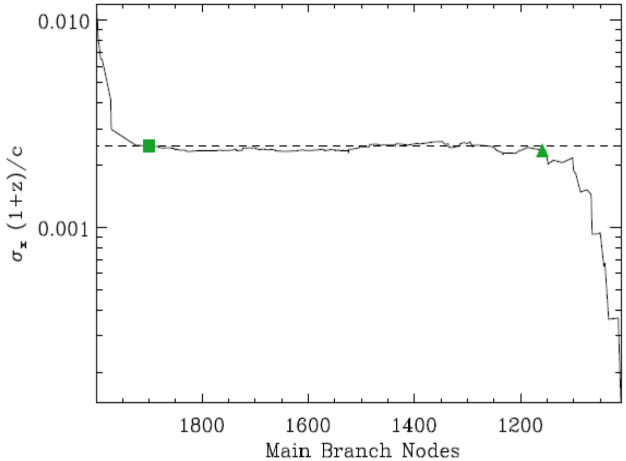
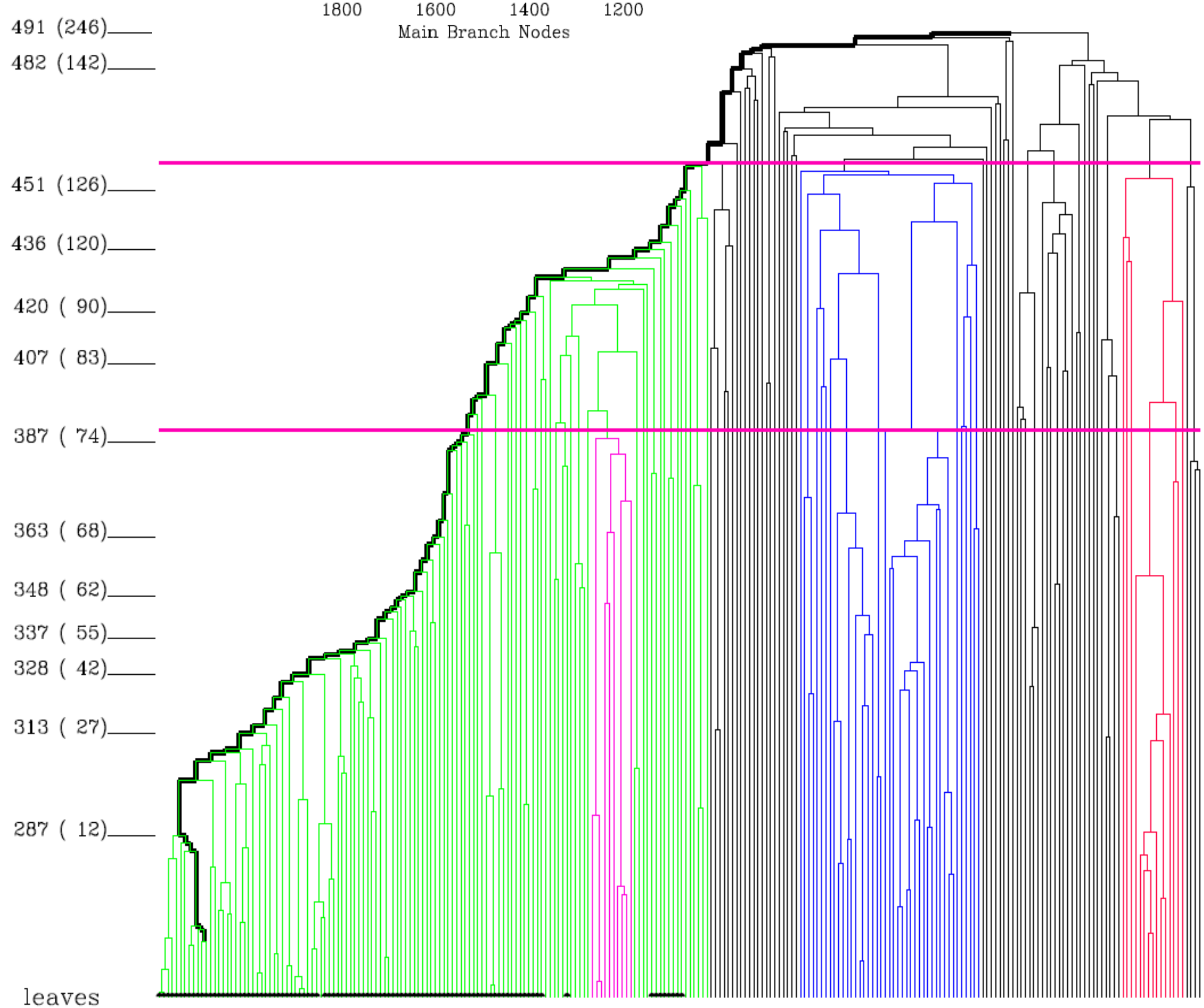
Binary tree & σ-plateau





Binding energy

↓

Binary Tree
- → **σ plateau**
- → **candidates to be members**
- → **center**

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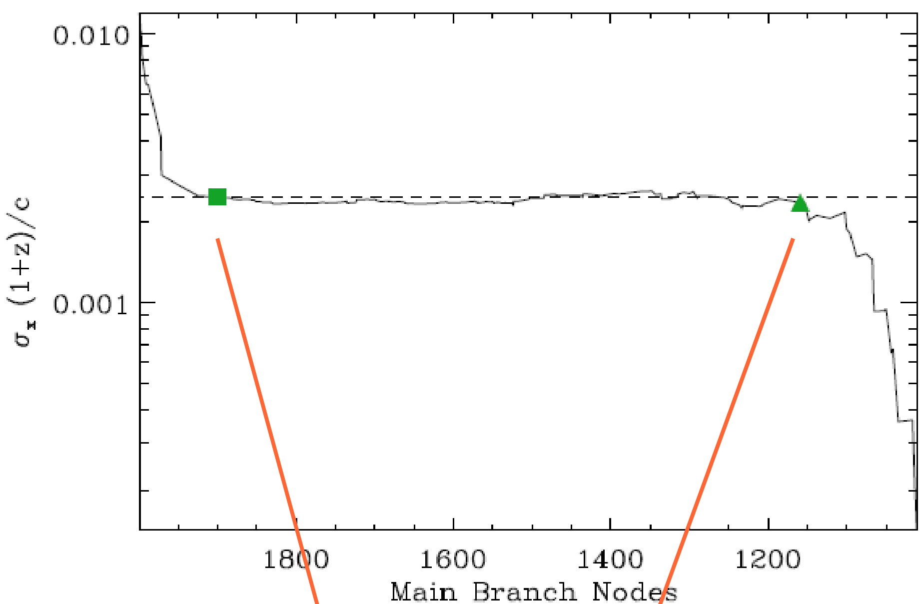
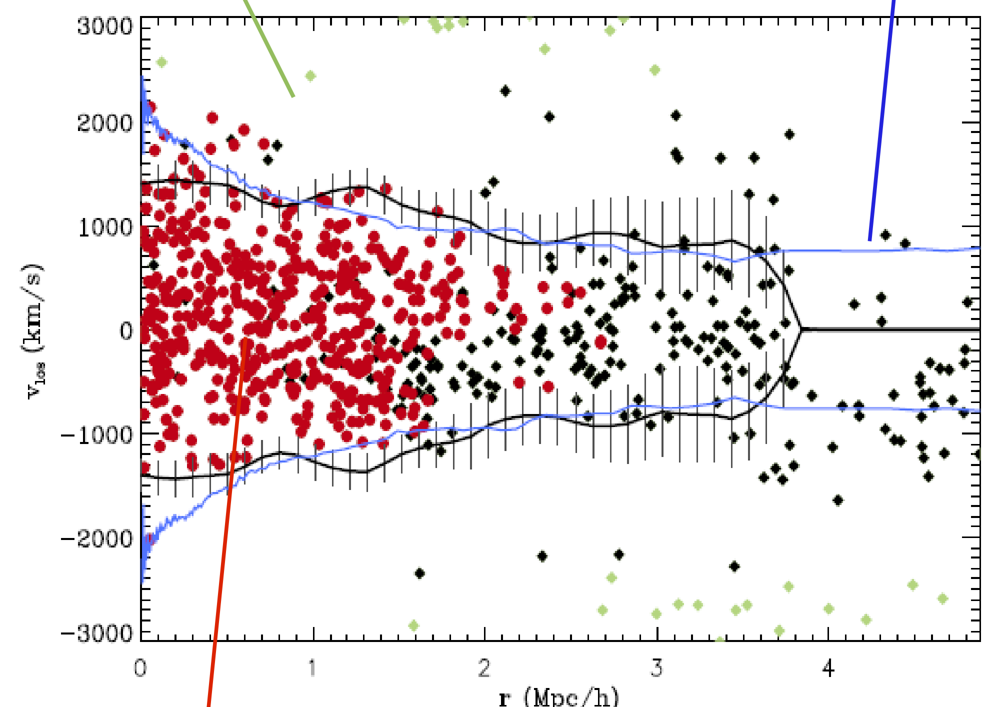
σ-plateau

Redshift diagram

true caustics

gxs outside 3.5 σ



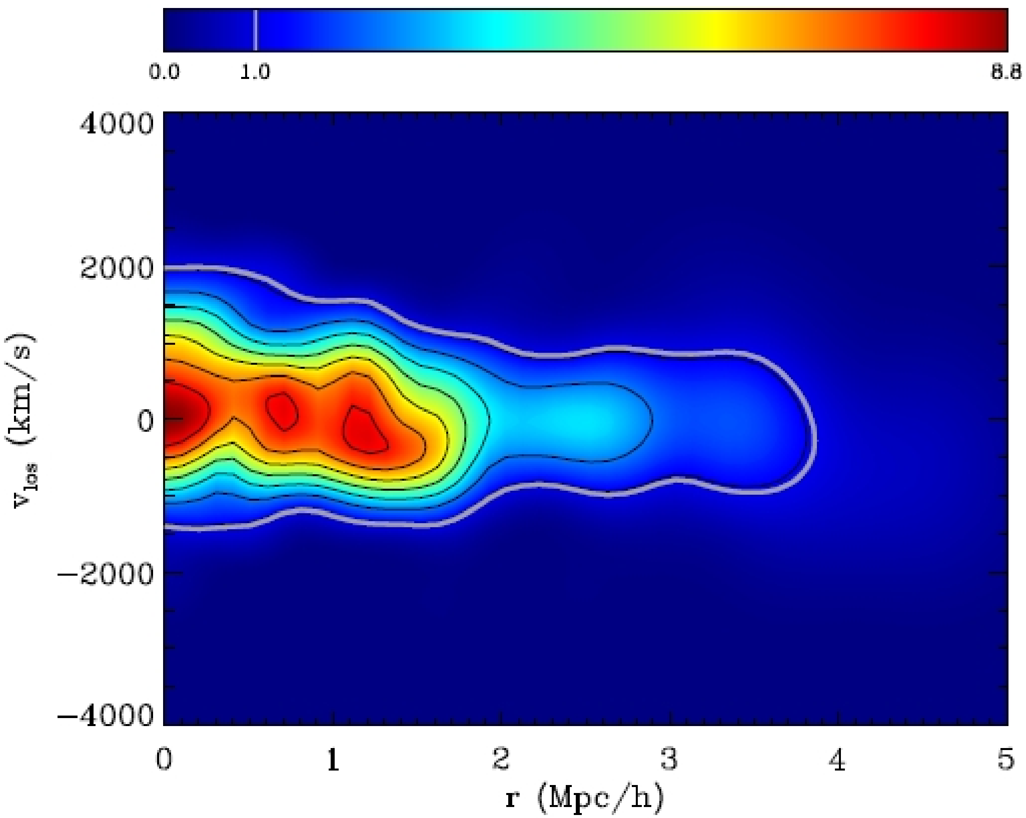


thresholds

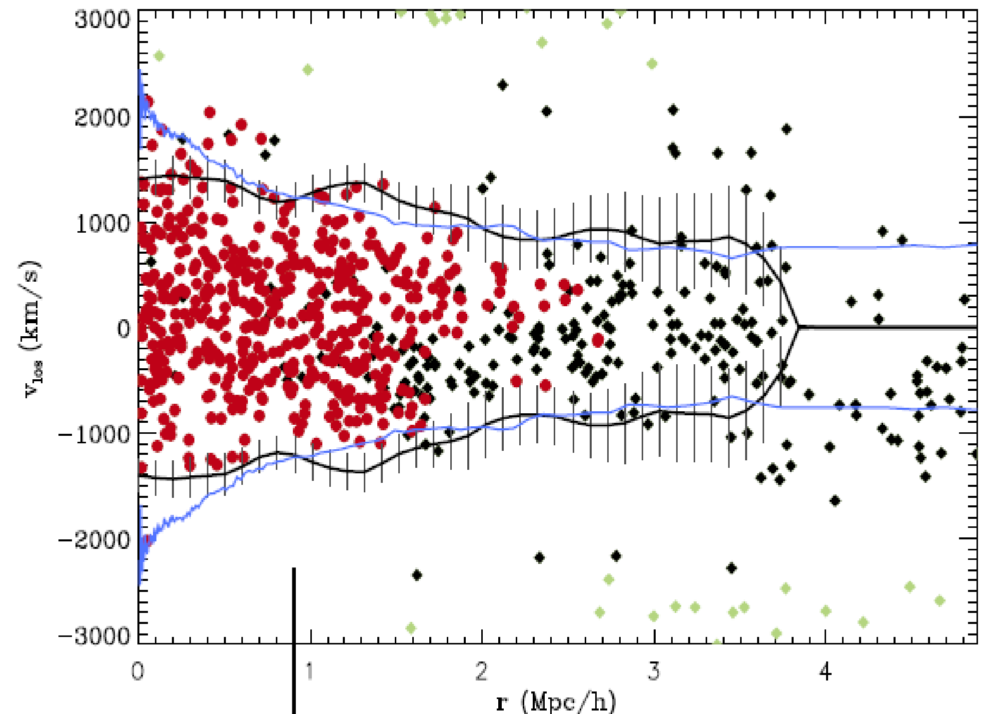
gxs with $r \leq 3R_{200}$

How to know which galaxies are real members

2D density f_q



Redshift diagram



If the cluster is isolated → f_q=0

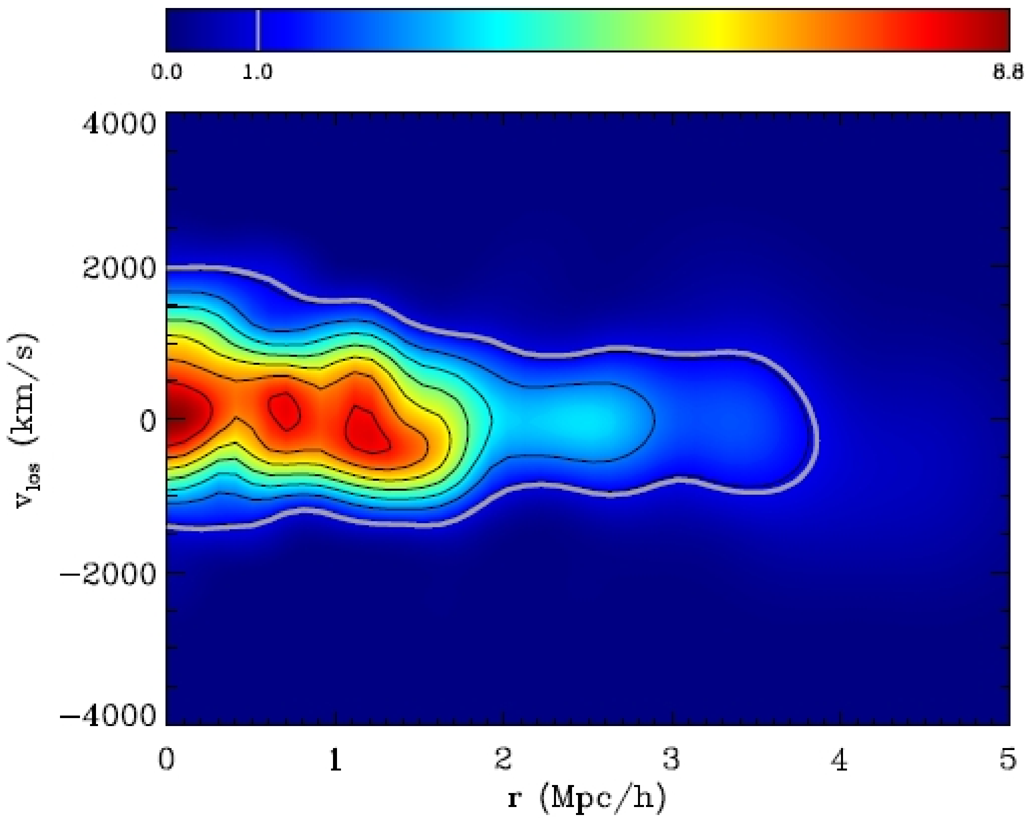
The distribution of N galaxies is described by the 2D function

$$f_q(\mathbf{x}) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{h_i^2} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h_i}\right)$$

$$\mathbf{x} = (r, v)$$

$$K(\mathrm{t}) = \begin{cases} 4\pi^{-1}(1 - t^2)^3 & t < 1 \\ 0 & \text{otherwise} \end{cases}$$

2D density f_q



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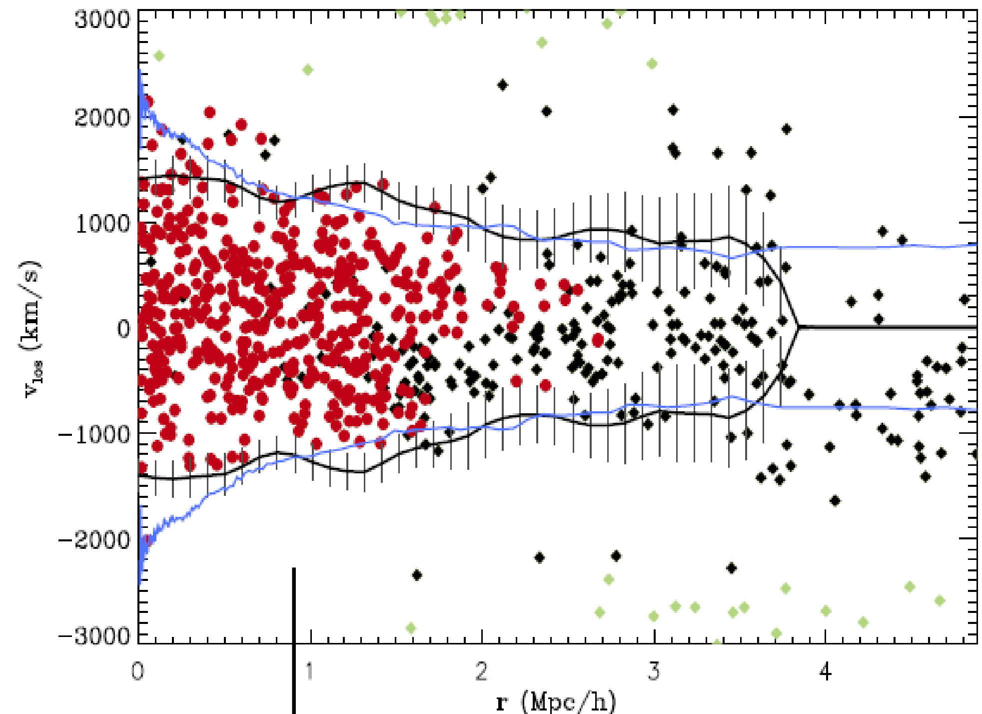

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If the cluster is ~~isolated~~ → f_q=0

We choose the parameter κ that determines the correct caustic location as the root of the equation

$$S(\kappa) \equiv \langle v_{\rm esc}^2 \rangle_{\kappa,R} - 4\langle v^2 \rangle = 0$$

When S(k)=0, the escape velocity inferred from the caustic amplitude equals the escape velocity of a system in dynamical equilibrium with a Maxwellian velocity distribution within R

$\langle v_{\rm esc}^2 \rangle_{\kappa,R} = \int_0^R \mathcal{A}_\kappa^2(r)\varphi(r){\rm d}r / \int_0^R \varphi(r){\rm d}r$ → Mean caustic amplitude within the main group size R

$\varphi(r) = \int f_q(r,v){\rm d}v$

$\langle v^2 \rangle^{1/2}$ → velocity dispersion of the candidate cluster members with respect to the median redshift

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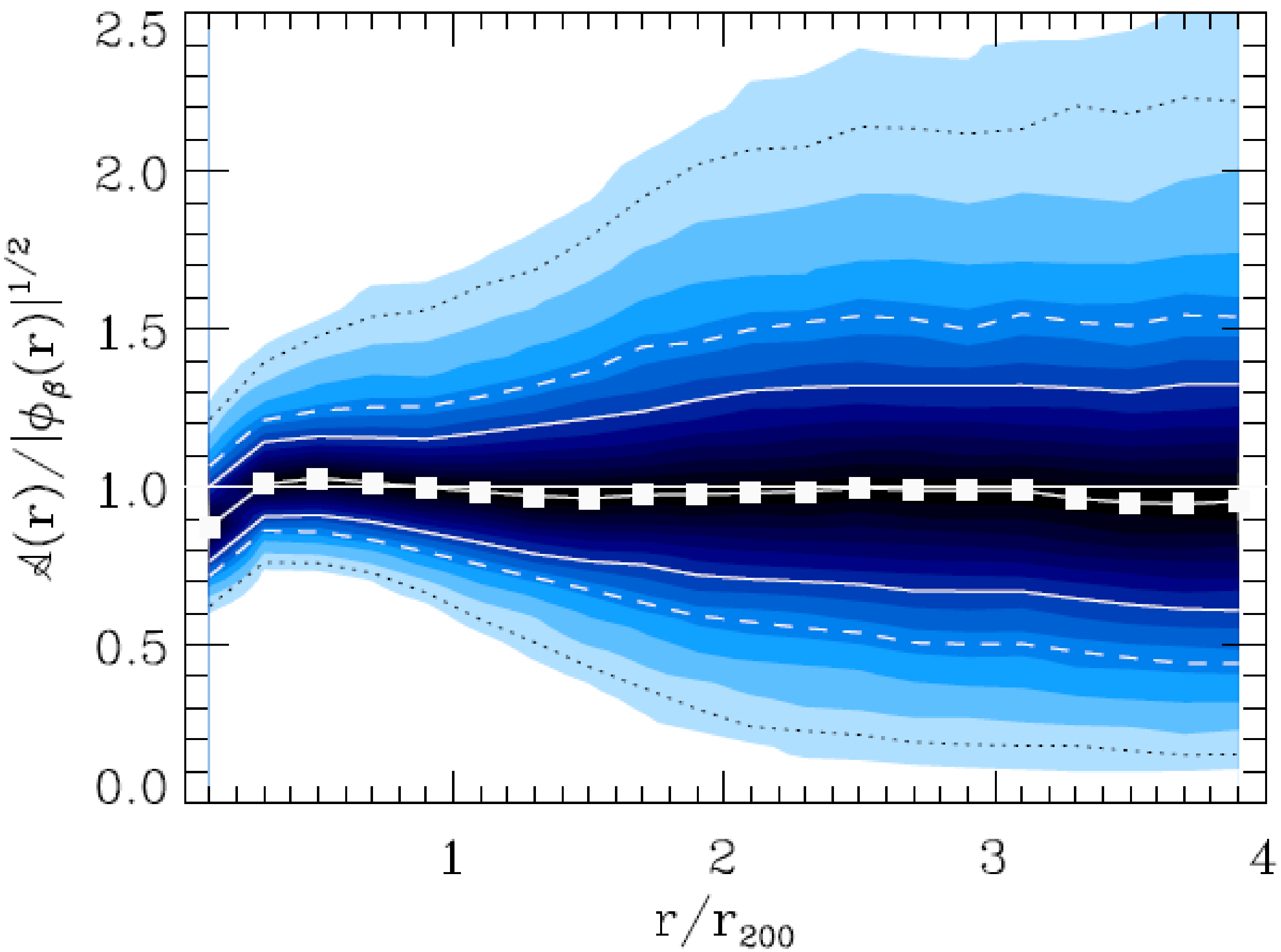
Applications

- Simulations
  - 1999- In most massive systems the mass is recovered within 20% out to 10 virial radii
  - 2010- Clusters with $M_{200} \geqslant 10^{14} h^{-1} M_{\odot}$
    - Escape velocity: better than 10% up to r~r_{200}



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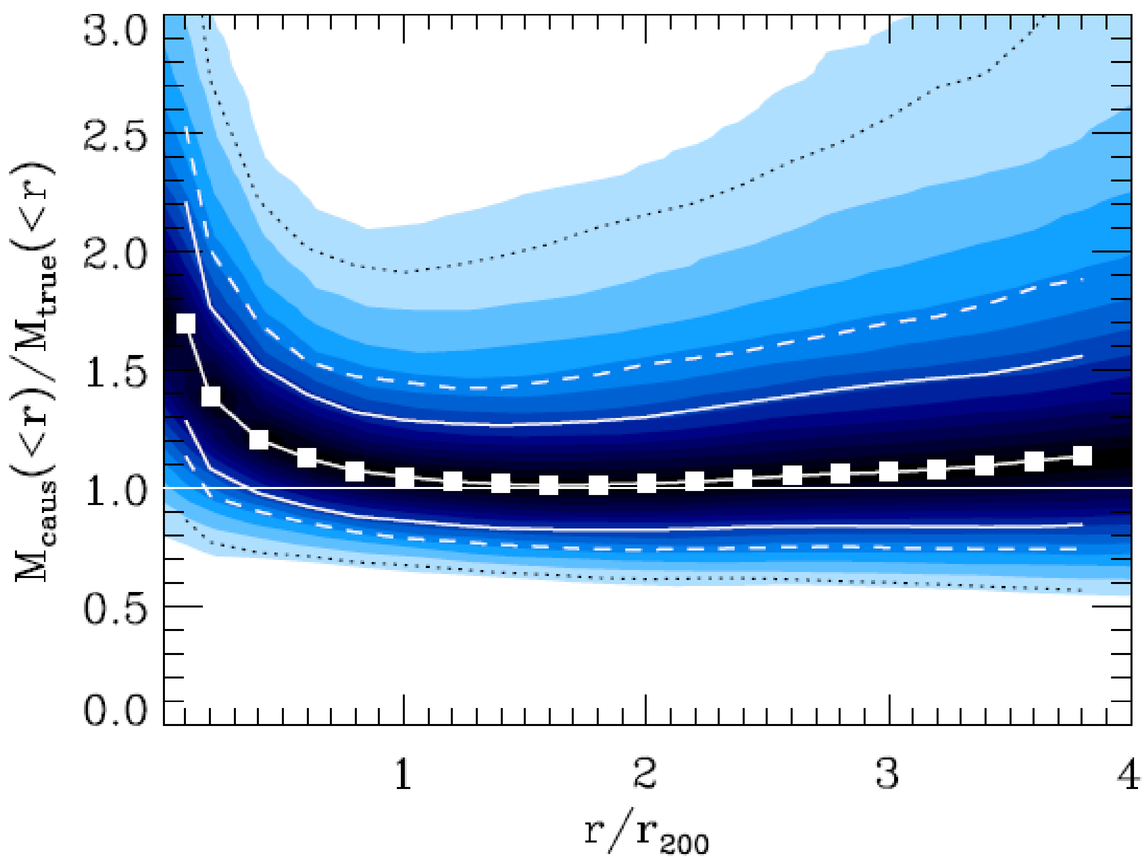
- Escape velocity: better than 10% up to r~r_{200}
- Mass profile: (0.6-4) r_{200} → 15% ; r<r_{200} → overestimation of the mass up to 70%



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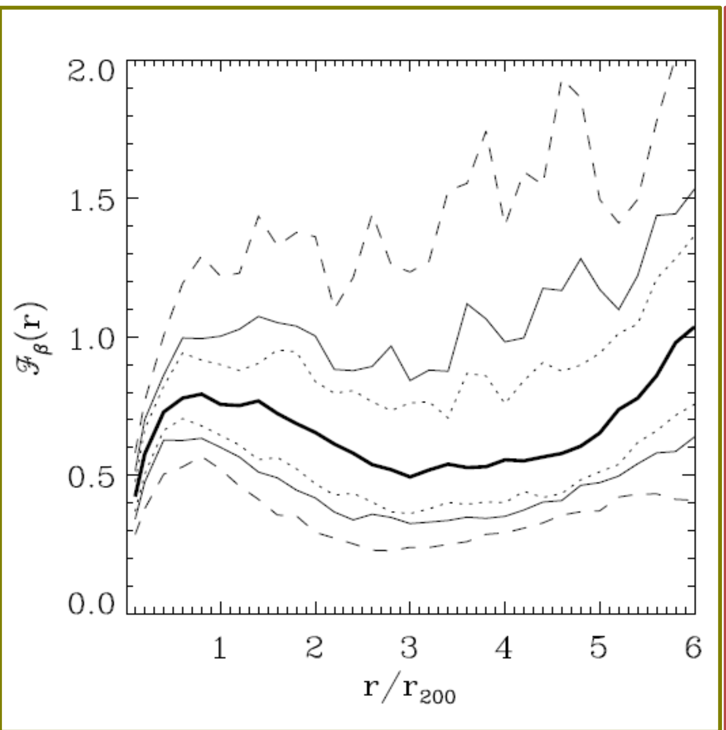
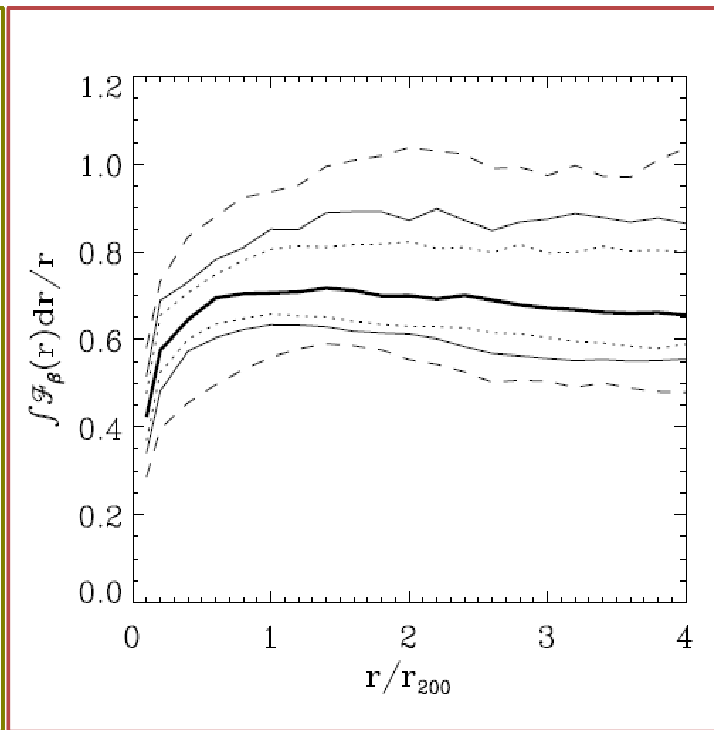
Because we neglect the radial dependence of $F_{\beta}(r)$





$F_{\beta}(r) \sim 0.7$

Applications

- Real systems
  - Coma (Geller et al. 1999) → NFW profile fits the cluster density profile
  - Cluster and Infall Region Nearby Survey (CAIRNS) (Rines et al 2003)
  - CIRS (Rines & Diaferio, 2006): 72 X-ray selected clusters with galaxy redshifts extracted from DR4-SDSS. Lasrgest sample of clusters whose mass profiles have been measured out to ~3r_{200} → virial mass function → cosmological parameters consistent with WMAP (Rines et al. 2007, 2008)
  - Groups of galaxies: 16 groups – NFW profile confirmed (Rines & Diaferio, 2008)
  - 43 stacked clusters from 2dF (Biviano & Girardi, 2003)
    → number of galaxies relatively small
  - Unrelaxed systems: Shapley superclusters (Reinsenegger et al. 2000, Davidzon et al. 2010?), Fornax cluster (Drinkwater et al. 2001), A2199 (Rines et al. 2002)
  - Dwarf spheroidal galaxies: only to determine members (Serra, Angus & Diaferio 201?)

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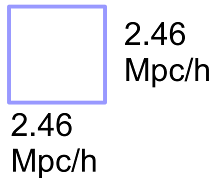
Disadvantages

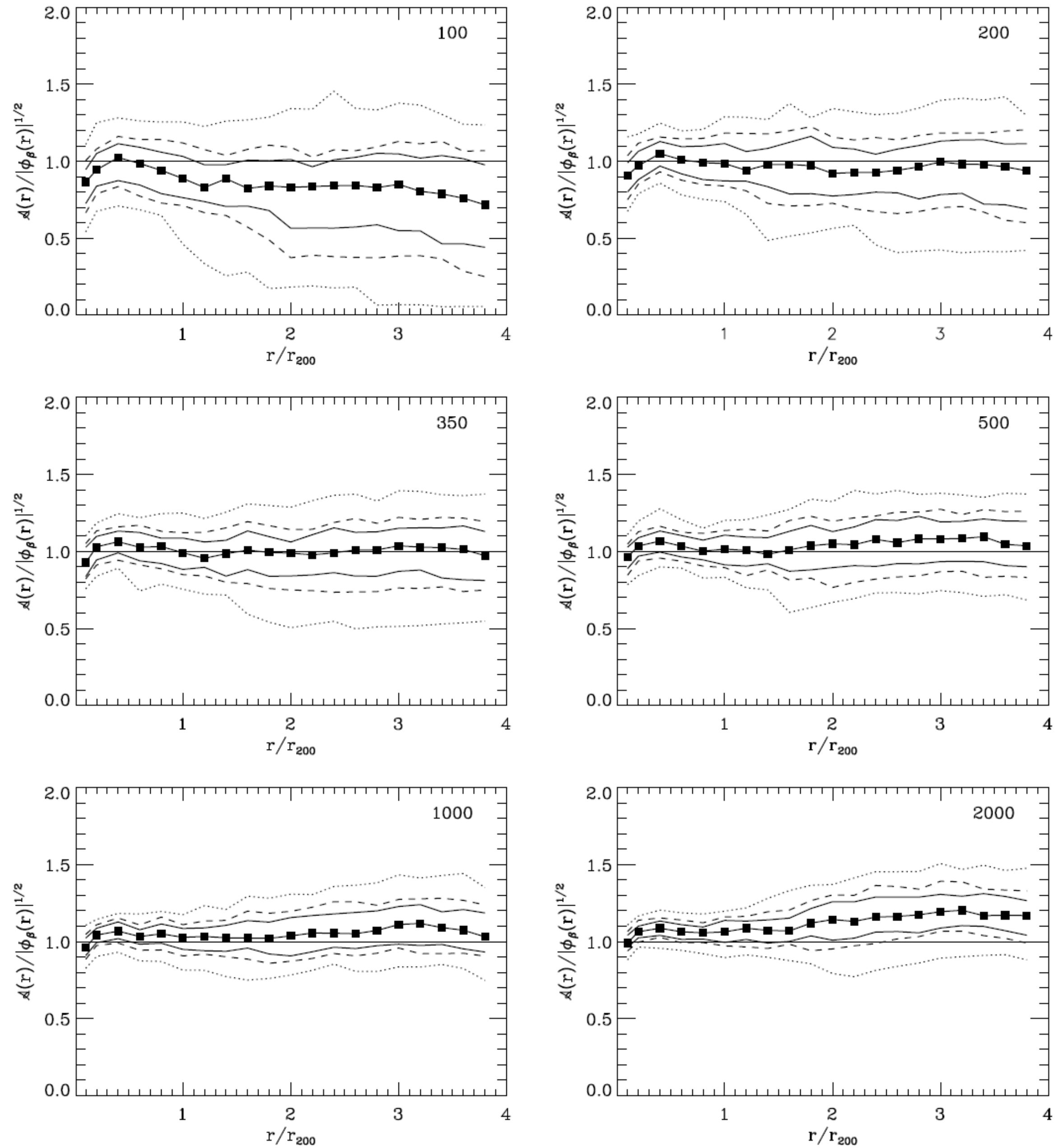
☆ Large number of redshifts needed

|Δv$_{los}$|=2000km/s



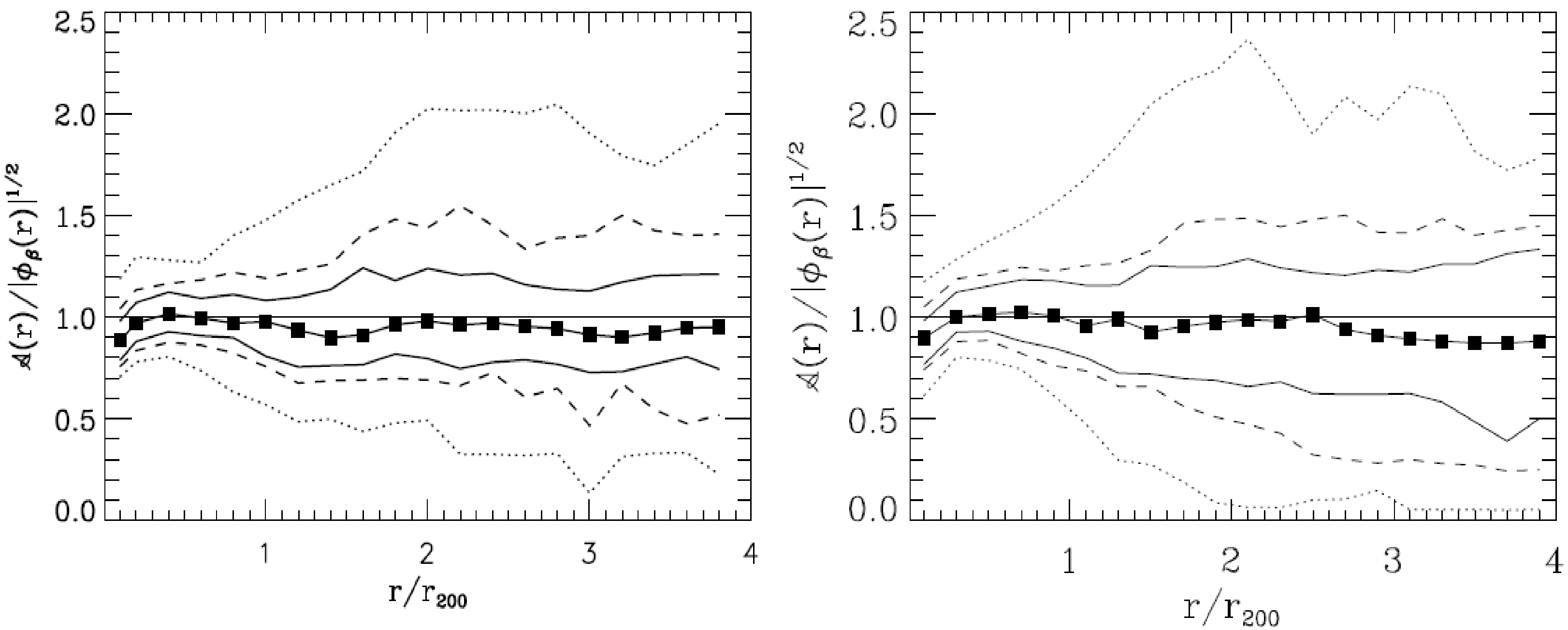
- Stacked clusters
- Spread decreases with number of galaxies
- Overestimation with rich catalogs

→ mock redshift catalogs from simulations show large scale structure less sharp than in real universe



Disadvantages

☆ Tunable parameters



• Scatter reduced
• Median unchanged

Byproducts

✯ Membership

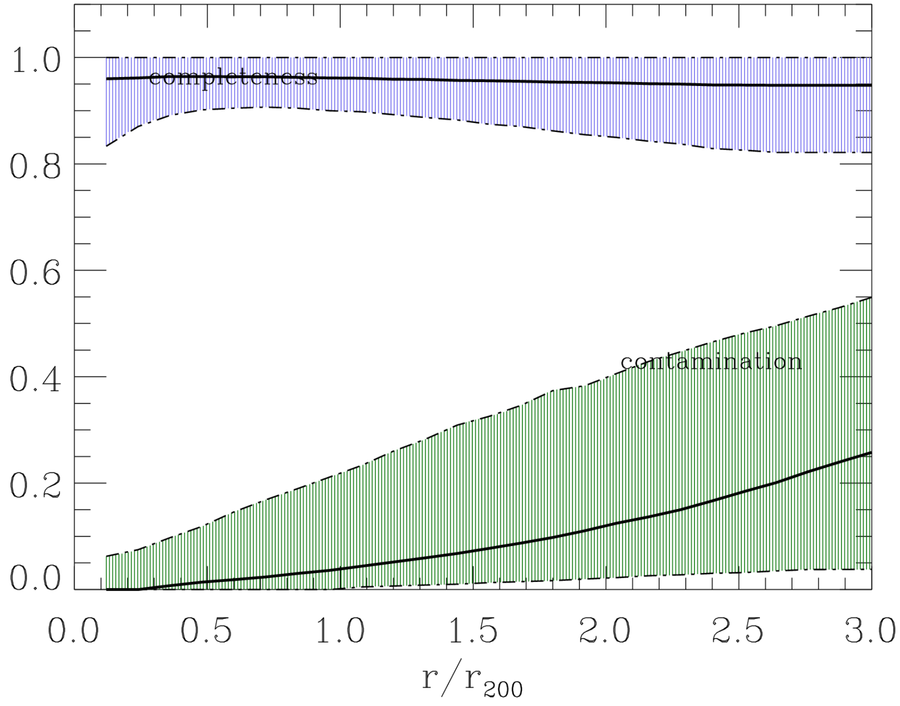
Escape velocity → members

Dwarf Spheroidal Galaxies

Mass → mond M/L lower than previous results

Clusters

To study the dependence of properties on the environment we need to know whether a galaxy is member of a cluster



If member is $r_{3D} \leq r_{200}$

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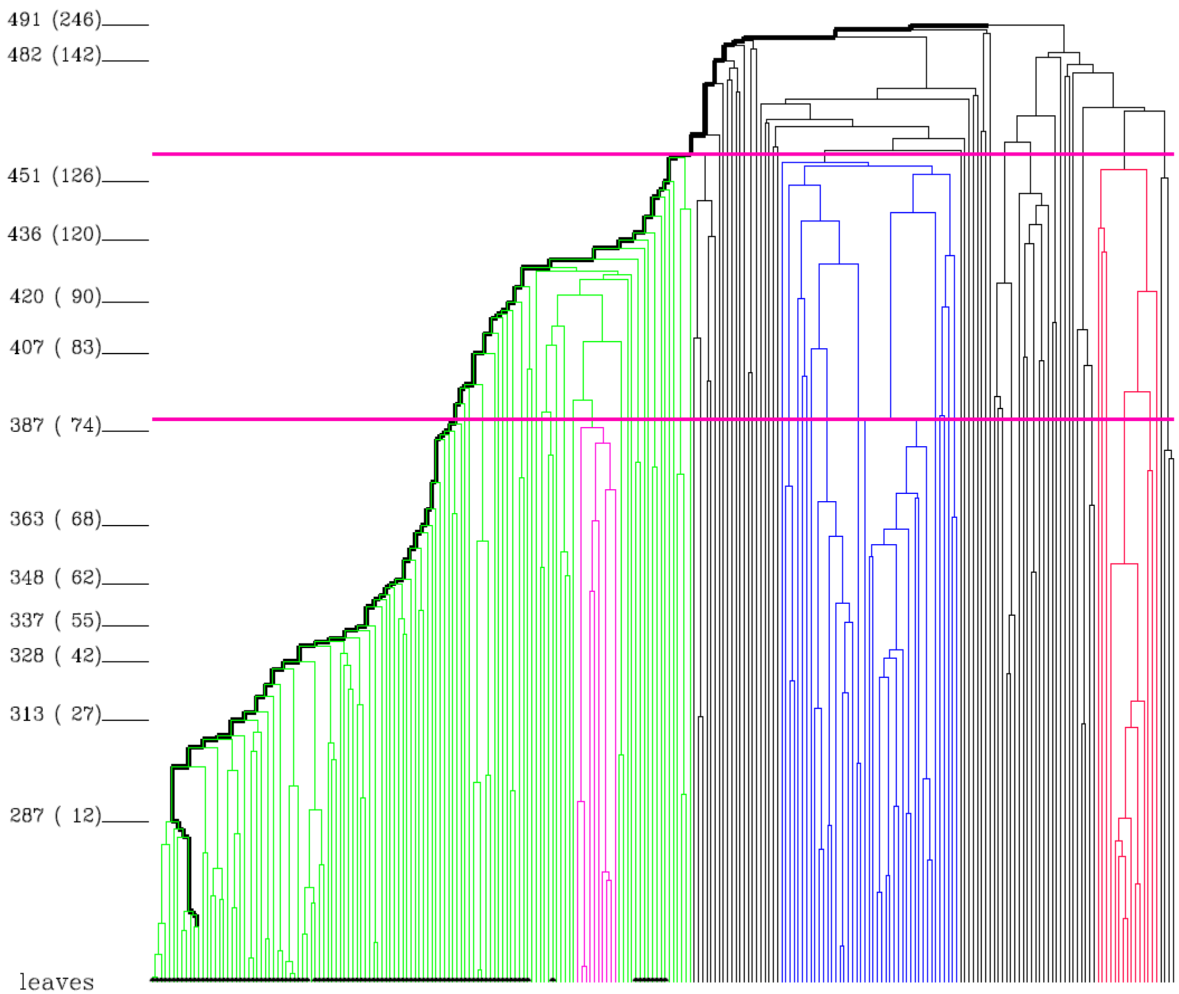


The binary tree is able to identify groups around the cluster and also substructures inside the cluster because it links the galaxies according to their pairwise projected binding energy

↓

FUTURE!

Conclusions

★ The Caustic Technique and gravitational lensing are the only 2 methods available to measure the mass profile of clusters beyond the virial radius without assuming dynamical equilibrium

★ The Caustic Technique in principle needs a dense redshift survey but ~2100 gxs in a field of 2.46 Mpc/h x 2.46 Mpc/h are enough to have an accurate escape velocity profile (although these results come from stacked clusters)

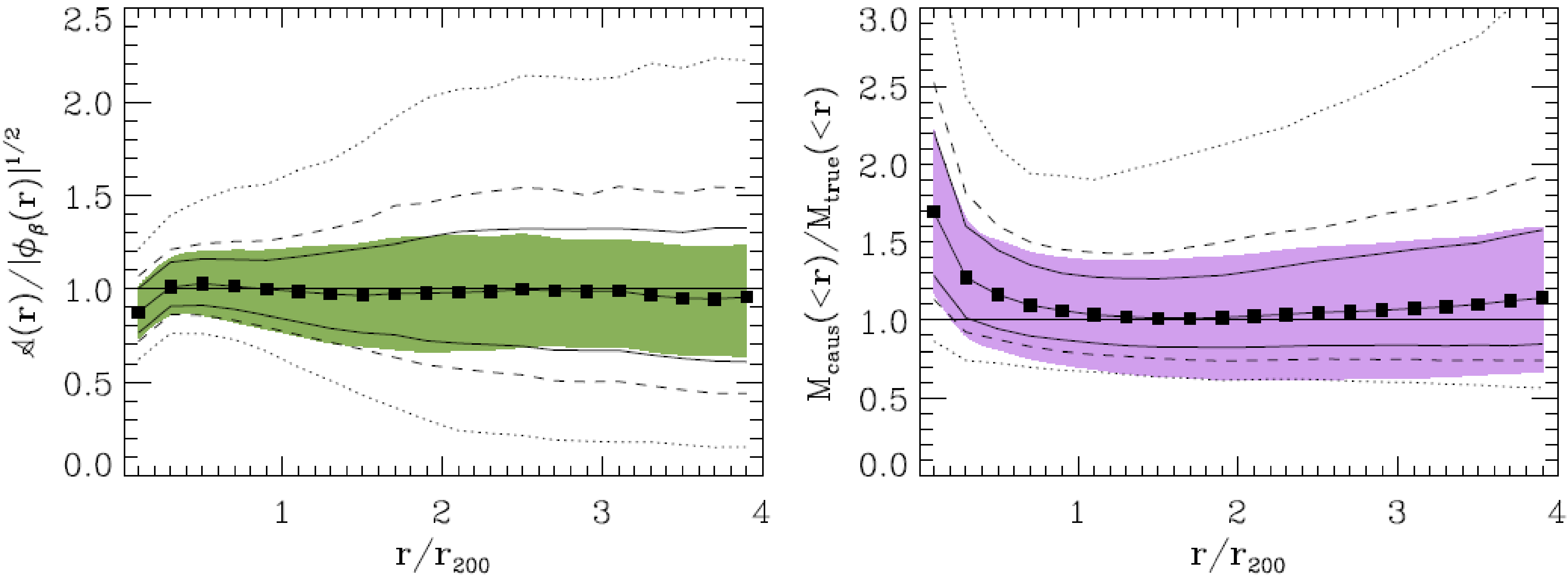
★ F_{β}(r) is not constant in the inner parts of the cluster. There is a sort of F_{β}-σ_{pl} degeneracy

→ If σ_{pl} is overestimated, the caustics would be wider and a lower value of F_{β} will be needed to obtain accurate mass profiles

★ Uncertainties due to projection → the same cluster when looked from another l.o.s. Gives different caustics, but the errors account for that

$$\delta M_i = \sum_{j=1,i} |2m_j \delta\mathcal{A}(r_j)/\mathcal{A}(r_j)|$$

Conclusions



⭐ The applications of the Caustic Technique to a large sample of simulated clusters demonstrated that the escape velocity is recovered with ~25% 1-σ uncertainty and the mass profile with ~50% 1-σ uncertainty

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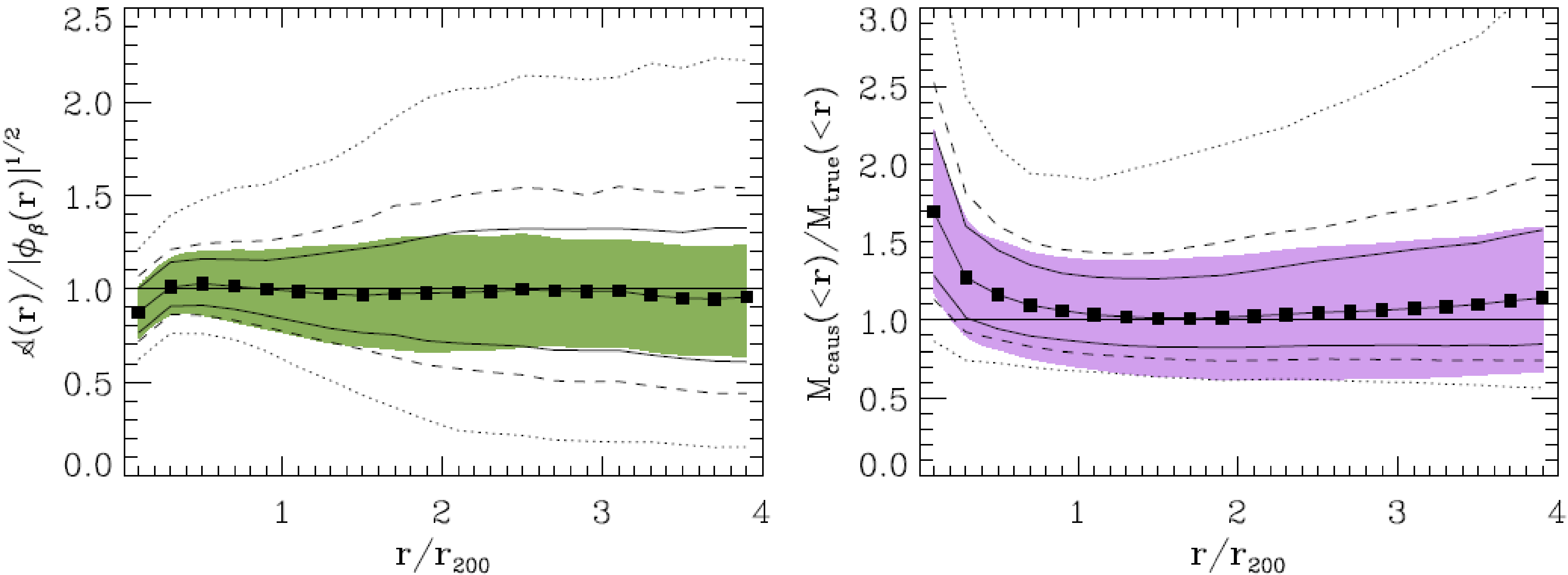

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THANKS!