Università degli Studi di Torino

CAUSTIC Journal Club

Galactic angular momenta and angular momentum correlations in the cosmological large-scale structure

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Stefano Camera

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UNIVERSITÀ DEGLI STUDI DI TORINO
ALMA UNIVERSITAS TAURINENSIS

Dipartimento di Fisica Generale "Amedeo Avogadro"
Università degli Studi di Torino
CAUSTIC
Conventional And Unconventional Studies & Tools In Cosmology

INFN Istituto Nazionale di Fisica Nucleare

Outline

Structure formation

Tidal torquing

Angular momentum distribution

Angular momentum correlation

Implications for weak gravitational lensing

Structure formation

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- $\delta(\mathbf{x}, a) = \left[\delta\rho(\mathbf{x}, a) - \rho_0(a)\right]/\rho_0(a)$ density contrast
- $H(a) = \dot{a}/a$ expansion history of the Universe
- $\Phi(\mathbf{x}, a)$ Newtonian potential

Structure formation

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Galactic angular momenta cannot arise from vortical initial perturbations in the velocity field.

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Lagrangian perturbation theory

$$\mathbf{x}(\mathbf{q}, t) = \mathbf{q} - D_+(t)\nabla\Psi(\mathbf{q})$$

$$\dot{\mathbf{x}}(\mathbf{q}, t) = -\dot{D}_+(t)\nabla\Psi(\mathbf{q})$$

- $D_+(t) = \delta(\mathbf{x}, a)/\delta(\mathbf{x}, 1)$ growth factor
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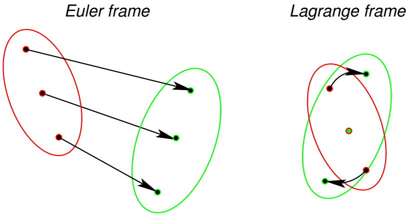

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$$= \frac{1}{2} a^2 \dot{D}_+ \left[\mathsf{I}, \mathsf{H}\bar{\Psi}\right]$$

- $\rho = \rho_0(1 + \delta) \simeq \rho_0$ because $\delta \ll 1$
- $\nabla\Psi \simeq \nabla\bar{\Psi} + (\mathbf{q} - \bar{\mathbf{q}})\mathsf{H}\bar{\Psi} \simeq (\mathbf{q} - \bar{\mathbf{q}})\mathsf{H}\bar{\Psi}$, with $\mathsf{H}\Psi$ the tidal shear
- $\mathsf{I} = \rho_0 a^3 \int \mathrm{d}\mathbf{q}\, (\mathbf{q} - \bar{\mathbf{q}})^T (\mathbf{q} - \bar{\mathbf{q}})$ inertia of the protogalactic cloud

$$\mathbf{X} \equiv \mathbf{I}\,\mathbf{H}\Psi$$

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Protogalactic objects will only acquire angular momentum if the eigensystems of their inertia and tidal shear are misaligned.

Angular momentum distribution

Galaxies form at local peaks in the large-scale structure by ellipsoidal collapse of the DM perturbation.

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$$n_p(\nu) = \int \mathrm{d}\mathbf{u}\, p(\mathbf{u})\mathcal{C}(\mathbf{u})$$

- $p(\mathbf{u})\mathrm{d}\mathbf{u}$ multivariate Gaussian random process
- $\mathcal{C}(\mathbf{u}) = \Theta\left[\delta - \nu\right]\delta_D\left[\nabla\delta\right]\Theta(-\lambda_i)|\lambda_1\lambda_2\lambda_3|$, with λ_i $(i = 1\ldots 3)$ eigenvalues of $\mathbf{H}\delta$

n-th moment of the angular momentum

$$\langle \mathbf{L}^n \rangle = \frac{1}{n_p(\nu)} \int \mathrm{d}\mathbf{u}\, p(\mathbf{u}) \mathcal{C}(\mathbf{u}) \mathbf{L}^n(\mathbf{u})$$

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The moments of the angular momentum (e.g. the variance) decrease if peaks of higher overdensity are selected.

Angular momentum correlation

Linear régime

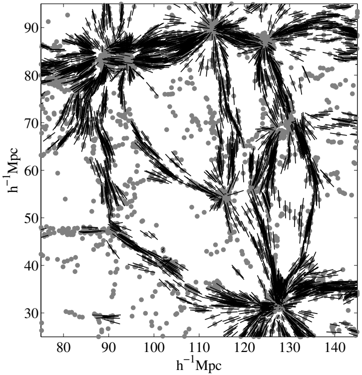
Correlation between neighbouring galaxies analytically via the covariance

$$C^{\mathbf{L}}(r) \equiv \mathrm{tr}\Big[\langle \mathbf{L}(\mathbf{x})\mathbf{L}^T(\mathbf{x}')\rangle\Big] =$$

$$= a^4 \dot{D}_+^2 \,\mathrm{tr}\Big[\langle \mathsf{X}(\mathbf{x})\mathsf{X}(\mathbf{x}')\rangle - \langle \mathsf{X}(\mathbf{x})\mathsf{X}^T(\mathbf{x}')\rangle\Big]$$

Non-linear régime

Correlation with the large-scale structure via N-body simulations



Hahn et al., 2007, MNRAS, 381, 41

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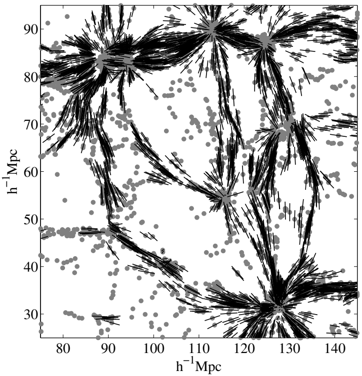

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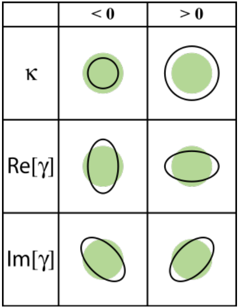
There is a correlation between galactic angular momenta (i.e. orientation of galaxies) and the large-scale structure.

Implications for weak gravitational lensing



$$\mathbf{A} \equiv \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa - \gamma_1 \end{pmatrix} = \int \mathrm{d}\chi \, W\left[\chi, n(\chi)\right] \nabla_\perp \Phi$$

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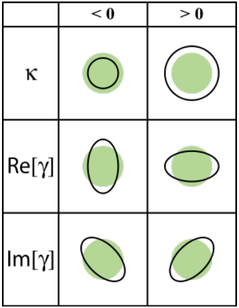

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Shear power spectrum

$$C^{\gamma\gamma}(\ell) = \frac{9}{16} {H_0}^4 {\Omega_m}^2 \ell^4 \int \mathrm{d}\chi \, W^2\left[\chi, n(\chi)\right] \frac{{D_+}^2(a)}{a^2} P^\delta\left(\frac{\ell}{\chi}, \chi\right)$$

- $\gamma = \gamma_1 + i\gamma_2$ complex shear
- $\langle \delta_k(z) \delta^\star_{k'}(z) \rangle = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') P^\delta(k, z)$

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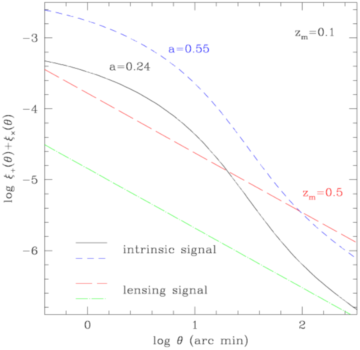

Crittenden et al., 2001, ApJ, 559, 552

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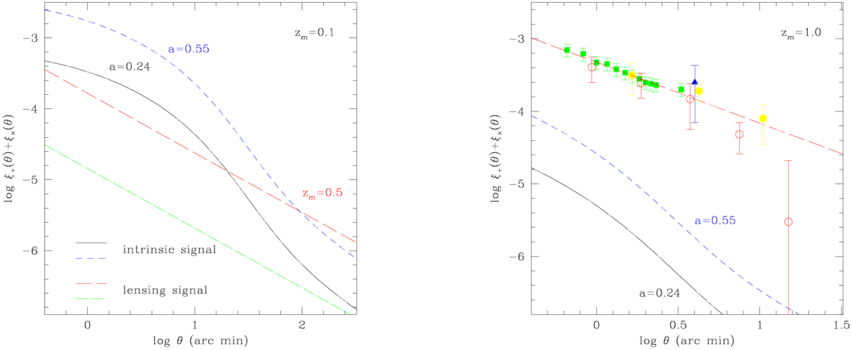

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Thanks for your attention!