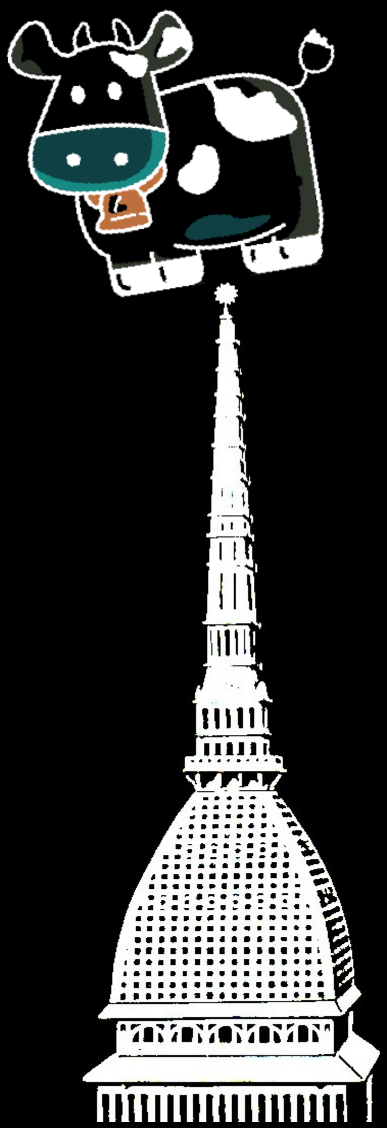


Journal Club

“Why all these prejudices against a constant ? ”

Bianchi & Rovelli 2010

(arXiv:1002.3966v3)

(organized by A. L. Serra)

Outline

- Paper's Abstract
- What do people say about Dark Energy?
- The three stories on the difficulties of the λ scenario
    - I. Einstein's field equations
    - II. Cosmic coincidence
    - III. The vacuum energy in quantum field theory
- Summary
- Conclusions

Paper's Abstract

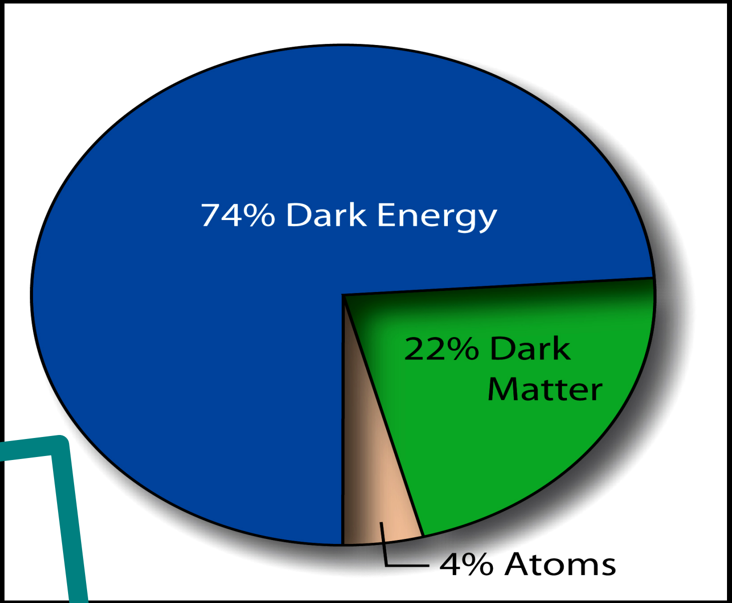
The expansion of the universe appears to be accelerating.

A simple explanation of this phenomenon is provided by the non vanishing of the cosmological constant in the Einstein equations.

Arguments are commonly presented to the effect that this simple explanation is not viable or not sufficient, and therefore we are facing the "great mistery" of the "nature of a dark energy".

We argue that these arguments are unconvincing, or ill-founded.

What do people say about Dark Energy ?



*" Arguably the **greatest mystery of humanity** today is the prospect that 75% of the Universe is made up of a **substance** known as "dark energy" about which we have almost no knowledge at all. "*

(Calder & Lahav 2010; Physics World 23)

What do people say about Dark Energy ?

Bianchi & Rovelli's objections:

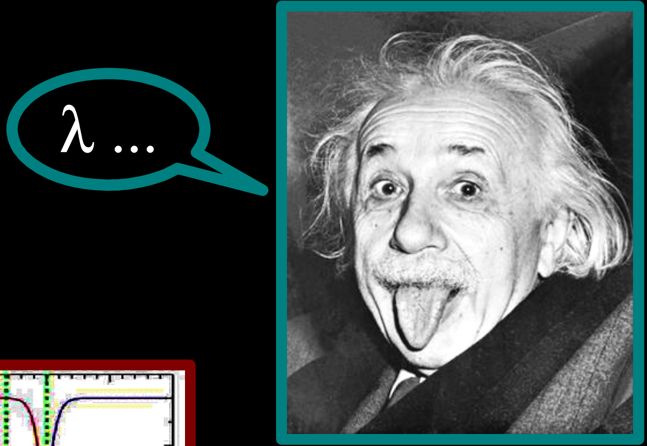
- ❐ There is no "great mystery" in the accelerated expansion of the Universe!
- ❐ The Dark Energy (DE) is not a "substance" ! (just as the centrifugal force is not!)
- ❐ Simple explanation for DE:

The acceleration of the Universe is a phenomenon which clearly predicted and simply described by the well-understood current physical theory (GR; λCDM)

➡ - Good to explore alternative theories
- Wrong to describe DE as "mystery" or "substance"

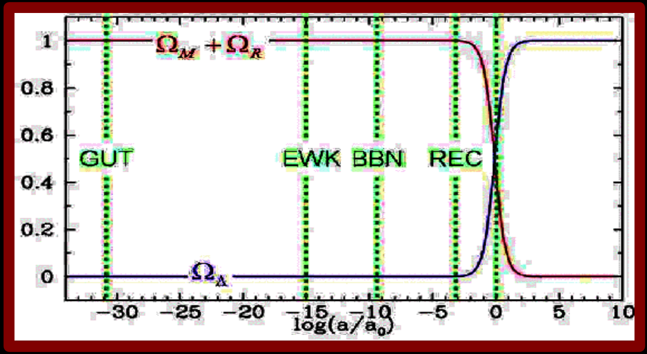
What do people say about Dark Energy ?

There are three stories routinely reported to highlight the difficulties of the λ scenario

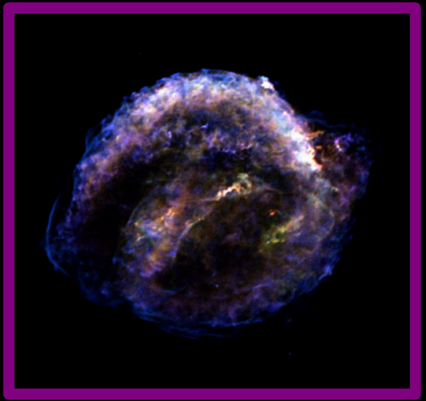


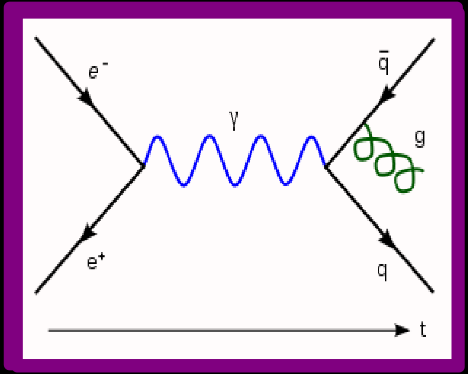
I. The rejection of the cosmological constant by Einstein

II. The coincidence problem



III. The huge difference between the small value of λ revealed by the cosmic acceleration and the large value of λ derived from QFT





Often presented in a confusing way!

Story n. I - Einstein's field equations

Story n. I - *Einstein's field equations*

Einstein, 1917: "*Cosmological Consideration in the General Theory of Relativity*"

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \quad \Rightarrow \quad R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Einstein, after 1929: "λ, my greatest blunder"

⇩

WHY ?

❐ Stories often told:

- Einstein unhappy about λ : it spoiled the beauty of GR ⇒ *nonsense !*
- he made a mistake he had to correct later ⇒ *many others during 1912-15 !*

❐ The "true" Einstein's blunder:

- he *did not believe his own equations*, predicting (in 1917 !) an expanding Universe
- he *forced these equations* to return a static Universe *in a wrong way* !

Story n. I - Einstein's field equations

Closed, homogeneous and isotropic Universe with radius *a*:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$ Einstein's equation

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\rho + \frac{\lambda}{3} - \frac{1}{a^2}$$ Friedmann's equation

Static Universe: $\dot{a} = \ddot{a} = 0 \quad \Leftrightarrow \quad \lambda = 4\pi G \rho = 1/a^2$

⇒ *Ad hoc* balance between λ, ρ, and *a* !

However: $\begin{cases} \rho = \rho_0 a^{-3} \\ a = a_0 + \delta a \end{cases}$

$$\frac{\ddot{a}}{a} = \frac{1}{3}(\lambda - 4\pi G \rho) \propto +(4\pi G \rho)\cdot \delta a$$

$$\Rightarrow \quad \frac{\ddot{a}}{a} = +\omega^2 \cdot \delta a$$

⇒ ***UNSTABLE EQUILIBRIUM!***

[Eddington, MNRAS, 1931]

Story n. I - *Einstein's field equations*

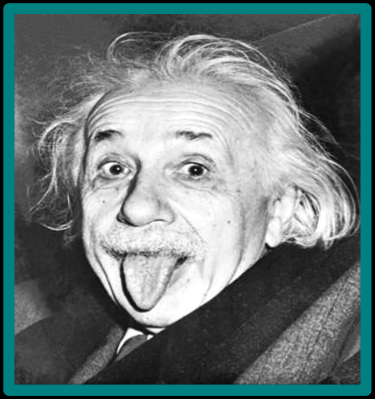
❒ Einstein's theory: prediction of the cosmic expansion/contraction

- WITHOUT λ
- WITH A GENERIC VALUE OF λ

and even

- WITH A FINE-TUNED VALUE OF λ (instability)

❒ Einstein's claim: equation with FINE-TUNED λ $\Rightarrow$ static universe



<u>Mistake on STABILITY to avoid a CORRECT PREDICTION !</u>

⇓

This was the "blunder"...
(not λ by itself)

Story n. I - Einstein's field equations

Einstein actually knew λ before his cosmological work

[Einstein, Ann. der Ph., 1916]

Most general 2nd order action for the gravitational field:

$$S[g]=\frac{1}{16\pi G}\int(R[g]-2\lambda)\sqrt{g}$$

Story n. I - Einstein's field equations

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[Einstein, Ann. der Ph., 1916]

Most general 2nd order action for the gravitational field:

$$S[g] = \frac{1}{16\pi G} \int (R[g] - 2\lambda)\sqrt{g}$$

Dependence on 2 constants: G, λ ⇒ no physical reason for discarding λ !

$\lambda=0$ *<u>more puzzling than</u>* $\lambda \neq 0$!

Gravitational physics: there is nothing mysterious in λ ...

....no more mystery than in any other physical constants!

Story n. I I - Cosmic coincidence

Story n. II - Cosmic coincidence

Generic cosmology:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

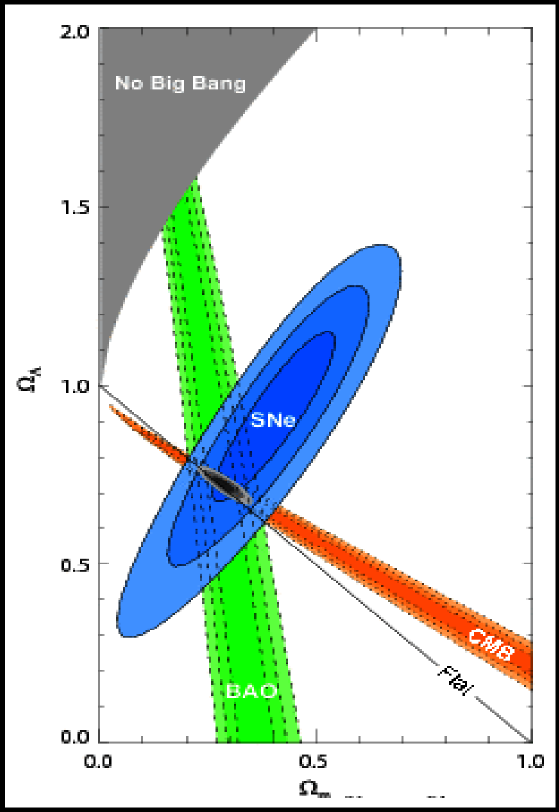
Einstein's equation with $\lambda \neq 0$

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\lambda}{3} - \frac{k}{a^2}$$

Friedmann's equation
(k=1,0,-1: closed,flat,open Universe)

Def. $\Omega_m = \frac{8\pi G \rho}{3H^2}$, $\Omega_\lambda = \frac{\lambda}{3H^2}$, $\Omega_k = \frac{-k}{a^2 H^2}$

$$\Omega_m + \Omega_\lambda + \Omega_k = 1$$



From observations: $\Omega_m + \Omega_\lambda \simeq 1$, $\Omega_k \simeq 0$

($\Omega_m + \Omega_\lambda = 1.054^{+0.048}_{-0.041}$ $\Rightarrow$ $k = +1$
Calder & Lahav 2010)

We will assume: $\Omega_m + \Omega_\lambda = 1$, $\Omega_k = 0$

Story n. II - Cosmic coincidence

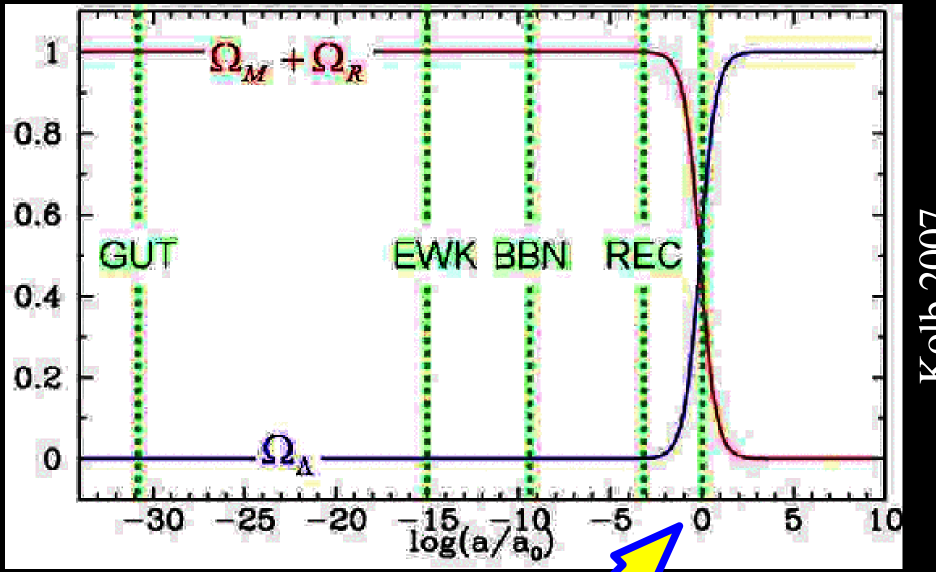
- **λCDM model:**

$$\Omega_m = \Omega_b + \Omega_{Dark\ Matter}$$

- **Observations:**

$$\Omega_b \simeq 0.0227 \pm 0.0006$$
$$\Omega_\lambda \simeq 0.74 \pm 0.03$$

$\Rightarrow$ $\Omega_\lambda \simeq 30\,\Omega_b \quad , \quad \Omega_\lambda \simeq 2.5\,\Omega_m$



Since:

$$\begin{cases} \rho_b(a) \simeq 0.0227\, a^{-3} H_0^2 \\ \rho_\lambda = const. \left(= \dfrac{\lambda}{4\pi G} \right) \end{cases}$$

then
1) **matter-dominated Universe** $\rho_b(a) \gg \rho_\lambda$
2) **"short" intermediate phase:** $\rho_\lambda \simeq \rho_b(a)$ $\Rightarrow$ **NOW !**
3) **λ-dominated Universe:** $\rho_\lambda \gg \rho_b(a)$

"Coincidence"

If we believe that the acceleration of the Universe is caused by λ, we have to believe that we live in a very special moment of the history of the Universe

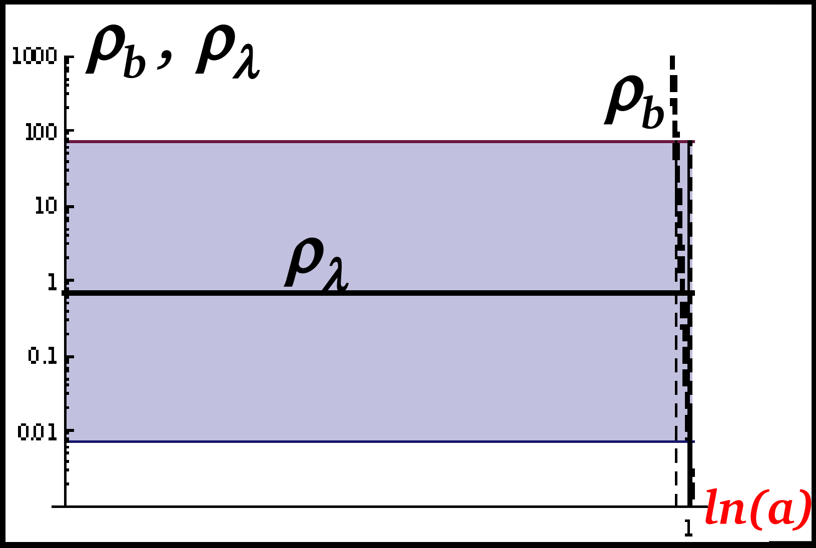
⇒ against the "cosmological principle"

Story n. II - Cosmic coincidence

Objections to the "coincidence" argument

1) <u>Equiprobability</u>

Universe expansion lasts forever in λCDM
⇒ we are NOT in a RANDOM moment, but
always **"AT THE BEGINNING" of $\Delta t \rightarrow \infty$**



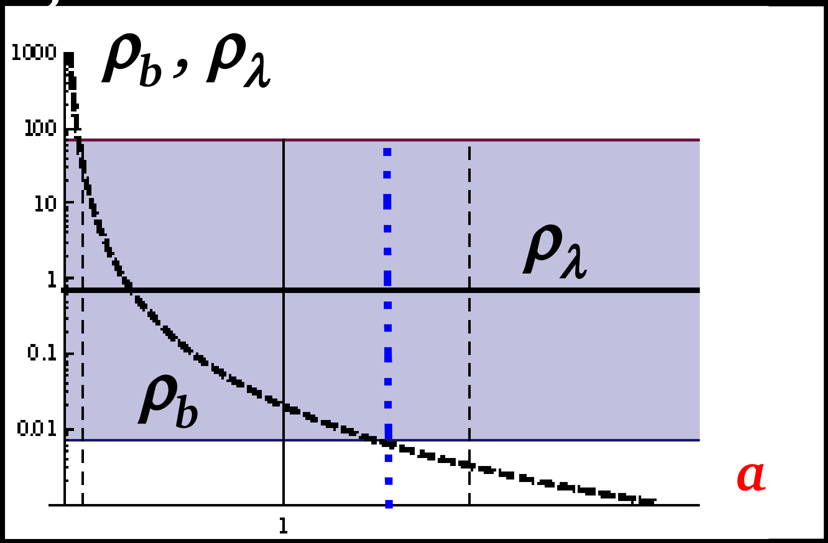
Appropriate question:
Are we in a special moment of $\Delta t \sim t_H$?
(e.g. from $t=0$ to $t=3t_H$)

▶ Logarithmic plots: YES!
(but why to use them?)

▶ **Linear** plots: **NO!**

$$\frac{\rho_\lambda}{\rho_b} \ , \ \frac{\rho_b}{\rho_\lambda} \sim 1-100$$

for most of the $\Delta t \sim 3t_H$!



⇒ Order-of-magnitude issue! No fine-tuning...

⇨ ***Why is t_H comparable with the time of the transition to a λ–dominated Universe?***

Story n. II - Cosmic coincidence

Objections to the "coincidence" argument

2) "Mild" anthropic principle

Application of the cosmological principle:
it can't be UNCRITICAL *[Dicke 1961, Carter 1973]*

Rigorous application ⇒ $\rho_{local} \sim \langle \rho_{Universe} \rangle$,
but we live in a high peak of the density fluctuations!

⇒ Mild form of the anthropic principle

"In order for us to comfortably exist and observe it, we must be in a galaxy and not in intergalactic space, and in a period of the Universe history where heavy elements abound but the Universe is not dead yet.
Since the Universe evolves, this period is not too long, on a logarithmic scale: it cuts out scales much shorter and much longer than ours. "

⇨ *In a Universe with our value of λ, it is reasonable that human beings exist during $\Delta t \sim 10^{10}$ yr where Ω_b and Ω_λ are within a few orders of mag from each other.*

Story n. II - Cosmic coincidence

Objections to the "coincidence" argument

1) **Equiprobability**

2) **"Mild" anthropic principle**

⇨ **The "coincidence" argument against λ is very weak and not rigorous:**

- **it cannot be applied to the future (∞)**
- **it cannot be extrapolated to the past, unless one assumes equiprobability in log scale (nonsense)**
- **it is based on a version of the cosmological principle which is known to fail on many other situations (e.g., density)**

Story n. I I I - The vacuum energy in QFT

Story n. III - The vacuum energy in QFT

Argument: *The quartic divergence of the vacuum energy density in QFT*

Short version

- QFT predicts a Planck-scale vacuum energy, which behaves as an effective cosmological constant.
  ⇒ QFT ***predicts*** the existence of a cosmological constant !

$$\lambda_{QFT} \sim \frac{c^4 M_P^2}{(h/2\pi)^2} \sim 10^{87} s^{-2}$$

- Observations imply:

$$\lambda \sim 10^{-35} s^{-2}$$

Prediction wrong by ~120 orders of magnitudes !
"The worst theoretical prediction in the history of physics"
[Hobson, Efstathiou & Lasenby 2006]

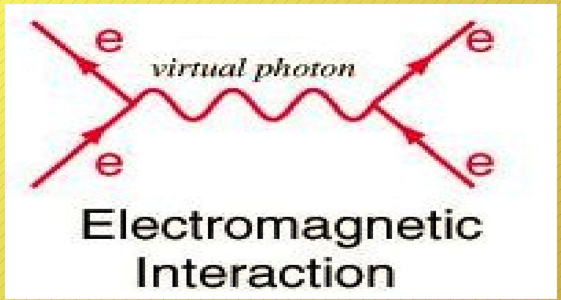
Story n. III - The vacuum energy in QFT

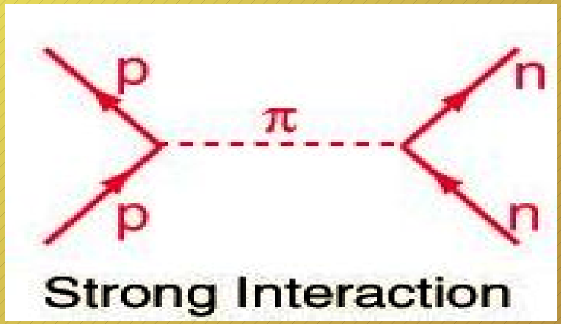
Basics of QFT

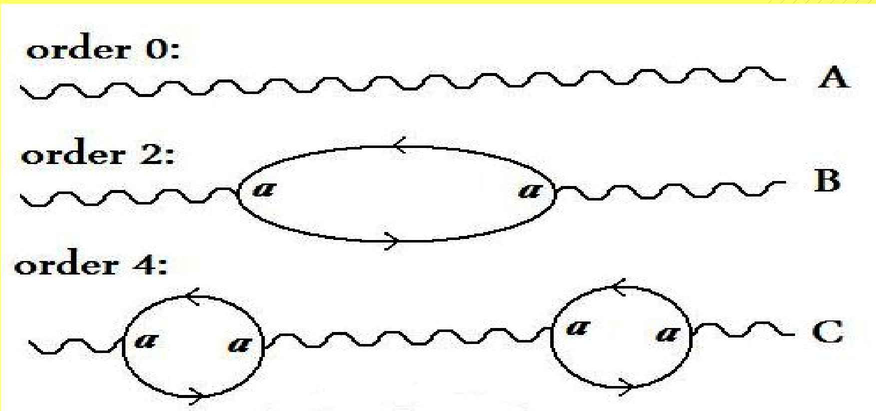
❒ ***QFT*: Theoretical framework to construct QM models of systems classically described by fields.**
Necessary at the confluence of Special Relativity with Quantum Mechanics

Particles=excited states of a field (field quanta)

Interactions between fermions described through exchange of virtual bosons, living $\Delta t \sim \hbar/m_b c^2$







❒ ***Perturbative QFT*: Technique used to describe complicated QFs in terms of a simpler one, for which a mathematical solution is known**
⇒ 1^{st}, 2^{nd} ...order loops

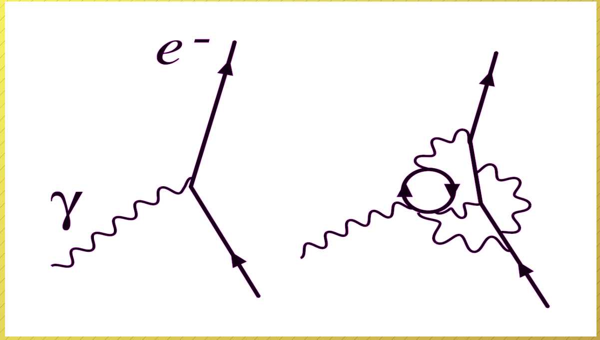
Story n. III - The vacuum energy in QFT

Basics of QFT

❐ *Divergence*: Many simple calculation involving sum over infinite number of energy levels yield divergent results

(E.g.: perturb. theory for the E-shift of an e^- in presence of a γ)

$$M = \int_0^\infty \Psi(k)\, d^4 k = \pm\infty$$



❐ *Renormalization*: Solution to the divergence problem

- Cutoff: quanta cannot have $k>k_{max}$

$$M = \int_0^{k_{max}} \ldots \in R$$

continuum space ⇒ *lattice* (no short λ)

(The FINITE high-energy mode contribution comes from a yet unknown theory)

- *Divergence* ⇒ *Cutoff-dependent* quantity $\quad M = M(k_{max})$
- *Cutoff-dependence* ⇒ dependence on an *observable* $\quad M = M(k_{obs})$

❖ "Renormalizable QFTs" :- those where $\mathcal{L}$'s constants diverge at worst as $\log(1/\kappa_{max})$

- the long-distance, low-E effective field theories for given short-distance, high-energy theories.
- insensitive to the precise nature of the underlying HE phenomenon (pros and cons)

Story n. III - The vacuum energy in QFT

Basics of QFT

❐ *Naturalness:* "Aestethic" criterion, according to which all the dimensionless ratios of the parameters of a theory should be ~ 1

" The smallness of a dimensionless parameter η *would be considered 'natural' only if a symmetry emerges in the limit* $\eta \to 0$ *"* *['t Hooft]*

All terms in the effective action that preserve the required symmetries should appear in this action with natural coefficients (unless a more detailed explanation exists)

Natural coefficients: $h = \eta\, K^{(4-d)}$,

d=dimension of the operator

K =cut-off (energy/length) scale, where the effective field theory breaks down

η=random number, not $\ll 1$ at K

❖ *Known SM problems:*

Higgs mass (hiearchy), Cosmological constant, Strong θ *angle (CP), ...*

Story n. III - The vacuum energy in QFT

Long version

A QFT with finite cut-off M: interpreted as an "*effective theory*", valid at scales ≪M, obtained by a HE theory integrating away the HE modes

Classical action describing the world:

$$S[\phi, g] = S_{SM}[\phi, g] + \frac{1}{16\pi G}\int (R[g] - 2\lambda)\sqrt{g}$$

ϕ : ensemble of the matter fields (fermions, gauge, Higgs)
g : metric

S_{SM} : Standard Model action on a curved spacetime

Cosmological constant

To study the corresponding quantum theory:

- expansion around a vacuum solution:

$$\begin{cases} \phi = \phi_0 + \delta\phi \\ g = \eta + h \end{cases}$$

(η : Minkowski metric)

- perturbative computation of the effective action:

$$\Gamma[\phi, g] = S[\phi, g] + \frac{h}{2\pi}\Gamma_1[\phi, g] + \dots$$

Story n. III - The vacuum energy in QFT

At one loop: $$\Gamma[\phi, g] = S[\phi, g] + \int \Lambda(M)\sqrt{g} + \ldots$$

$$\Rightarrow \quad \Lambda(M) = O(L^{-4}) = O(M^4)$$

This term renormalizes λ in the action $S[\phi, g]$:

$$\frac{\lambda}{8\pi G} \Rightarrow \frac{\lambda}{8\pi G} + \Lambda(M) \equiv \frac{\lambda_{QFT}}{8\pi G}$$

Taking a cut-off

- ~ Planck mass M_P (~10^{19} GeV) $\Rightarrow \lambda_{QFT} \sim \frac{c^4 M_P^2}{(h/2\pi)^2} \sim 10^{87}\ s^{-2}$ — 10^{120} !
- $\ll M_P$; e^- pointlike, $M \sim 10^2$ GeV $\Rightarrow \lambda_{QFT} \sim 10^{20}\ s^{-2}$ — 10^{55} !

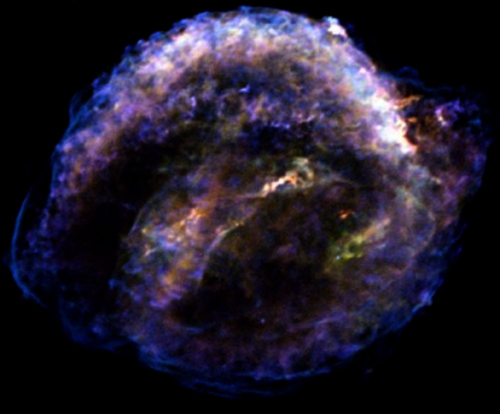
- QFT : no known mechanism protects λ from the scaling
- "Naturalness": there should be such a mechanism!

→ we might be missing something crucial … (cfr. Higgs)

Story n. III - The vacuum energy in QFT

We actually have ***TWO INDEPENDENT*** problems

1) A "low-energy gravitational physics" problem:



Q: In the classical Einstein equation,
$\lambda=0$ or $\lambda\neq 0$?
A: Observations say: $\lambda\neq 0$!

2) A "high-energy particle physics" problem:

Q: In QFT, is there a mechanism protecting λ from scaling so much*?*
A: N/A! ($\lambda\neq 0$ from obs. doesn't help...)

Story n. III - The vacuum energy in QFT

***Q*:** **Why so much confusion, then?**

***A*:** **Because there are *two ways* of viewing the cosmological constant in the action**

(1) To assume that *λ=0* in the gravitational lagrangian

$$\begin{cases} S[\phi, g] = S_{SM}[\phi, g] + \dfrac{1}{16\pi G}\displaystyle\int (R[g] - \cancel{2\lambda})\sqrt{g} \\ \cancel{\dfrac{\lambda}{8\pi G}} \Rightarrow \cancel{\dfrac{\lambda}{8\pi G}} + \Lambda(M) \equiv \dfrac{\lambda_{QFT}}{8\pi G} \end{cases}$$

and that the accelerated expansion is ENTIRELY due to *Λ*

Story n. III - The vacuum energy in QFT

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and that the accelerated expansion is ENTIRELY due to Λ

(2) To assume that there is a term $\lambda \neq 0$ in the gravitational lagrangian

$$S[\phi, g] = S_{SM}[\phi, g] + \frac{1}{16\pi G}\int (R[g] - 2\lambda)\sqrt{g}$$

which might or might not be renormalized through radiative corrections

Story n. III - The vacuum energy in QFT

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and that the accelerated expansion is ENTIRELY due to *Λ*

(2) To assume that there is a term *λ≠0* in the gravitational lagrangian

$$S[\phi, g] = S_{SM}[\phi, g] + \frac{1}{16\pi G}\int (R[g] - 2\lambda)\sqrt{g}$$

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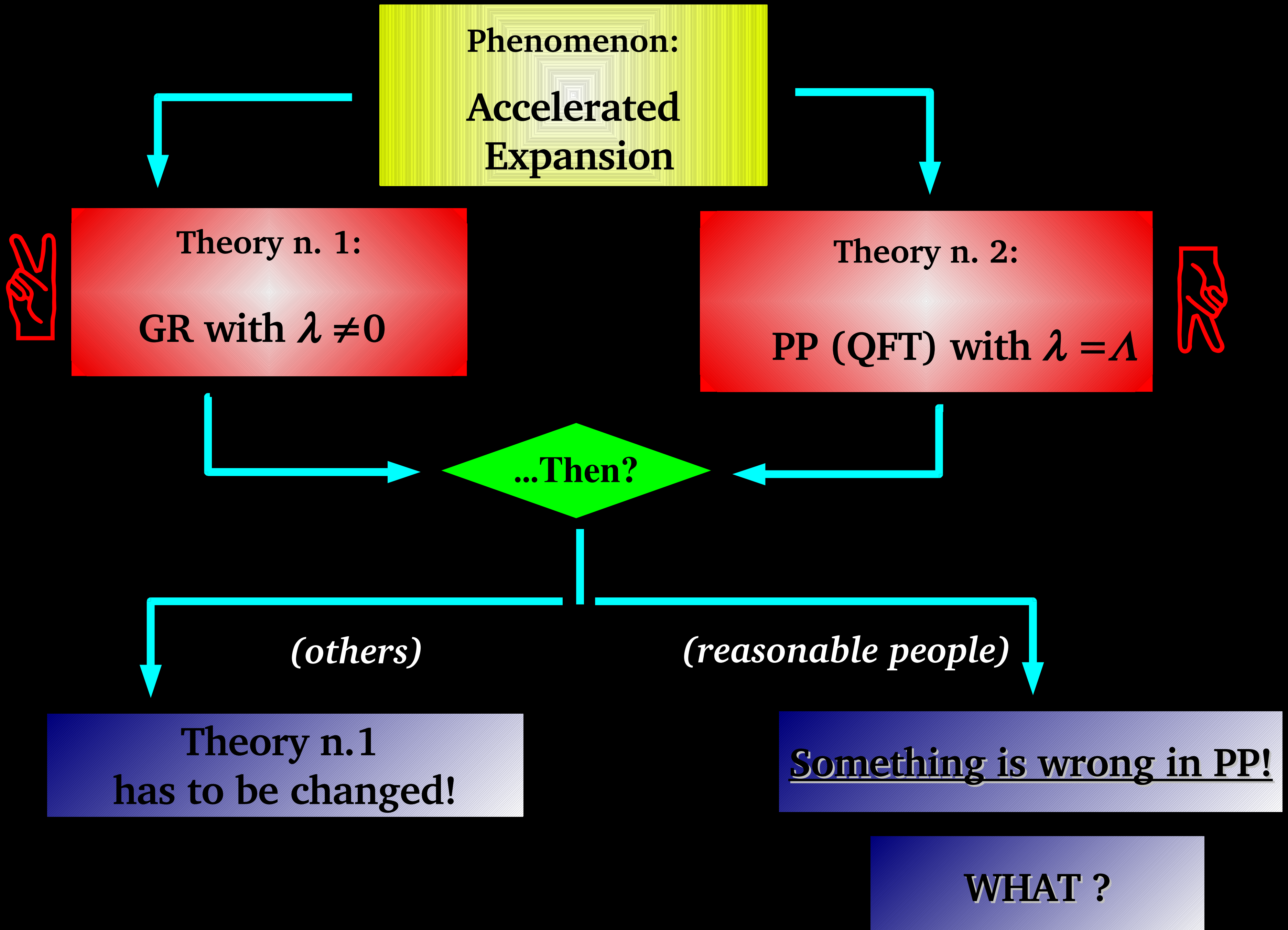
Story n. III - The vacuum energy in QFT



Story n. III - The vacuum energy in QFT

WHAT ?

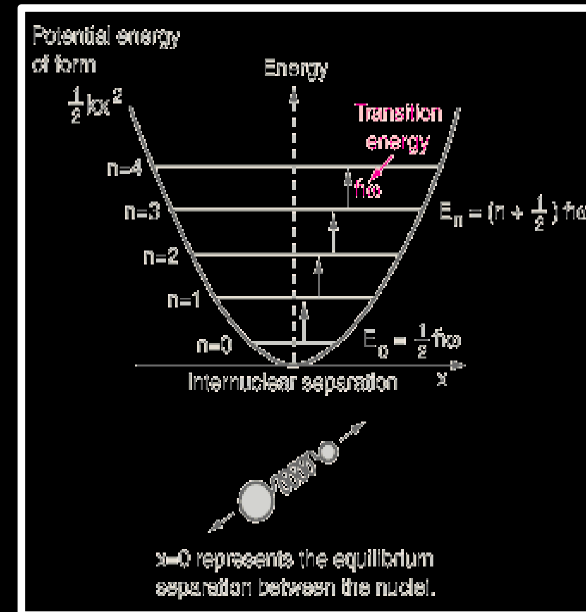
Possible problems in PP:

i. *Reality of the huge vacuum energy predicted by QFT*

ii. *Consistency of the theoretical context*

Story n. III - The vacuum energy in QFT

i. *Reality of the huge vacuum energy predicted by QFT*

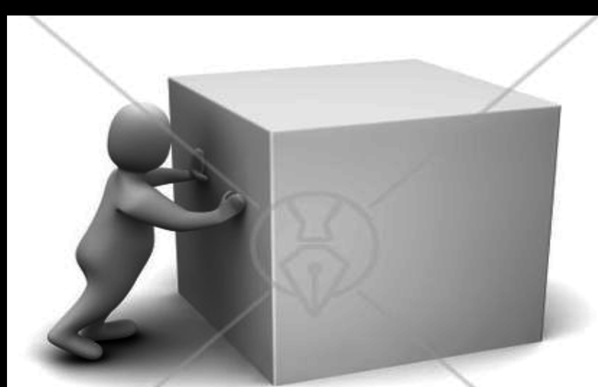


- Harmonic oscillator of frequency: ω

Vacuum energy: $E_0 = \frac{1}{2}\frac{h}{2\pi}\omega$

- Free field in a box:
collection of oscillators of freq. ω_k
per oscillation mode *k* (∃ ∞ *k*)

Vacuum energy: $E_0 = \frac{1}{2}\frac{h}{2\pi}\sum_0^\infty \omega_k = \infty$



- **Does this energy exist?**
- **Does it gravitate ?**

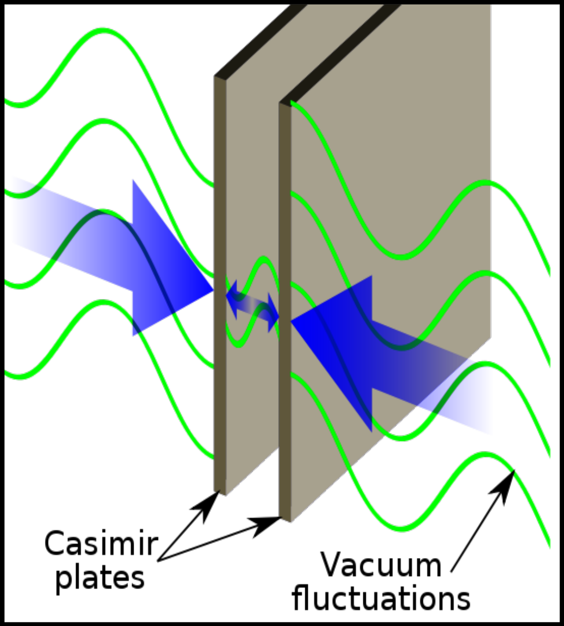
Story n. III - The vacuum energy in QFT

i. *Reality of the huge vacuum energy predicted by QFT*

NO ! *VACUUM ENERGY DOES NOT GRAVITATE*

Casimir effect

Reveals only the "change" in
vacuum energy, not the zero point !



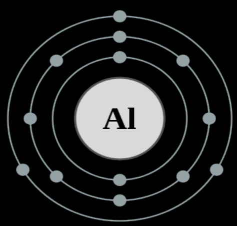
"No known phenomenon, including the Casimir effect, demonstrates that zero-point energies are real" [Jaffe 2005]

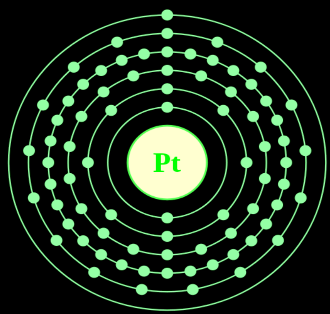
Story n. III - The vacuum energy in QFT

i. *Reality of the huge vacuum energy predicted by QFT*

BUT…

A SHIFT IN VACUUM ENERGY DOES GRAVITATE

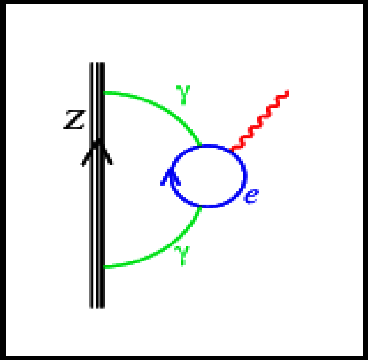




$E_{es} \sim 10^{-3}\, M_{tot,in}$ $\quad$ $E_{es} \sim 3 \cdot 10^{-3}\, M_{tot,in}$

Al, Pt: $\quad M_{grav}/M_{in} \sim 1+10^{-12}$

[Dicke]



e^- loop:

$$\Delta E_{es} \sim 10^{-3}\, E_{es}$$

[Polchinski 2006]

Al: $\quad \Delta E_{es} = 10^{-3}\, E_{es} = 10^{-3} \cdot 10^{-3}\, M_{tot,in} = 10^{-6}\, M_{tot,in} = 10^{-6}\, M_{tot,grav}$

Pt: $\quad \Delta E_{es} = 10^{-3}\, E_{es} = 10^{-3} \cdot 3 \cdot 10^{-3}\, M_{tot,in} = 3 \cdot 10^{-6}\, M_{tot,in} \sim 3 \cdot 10^{-6}\, M_{tot,\, grav}$

we know to a high precision ($1/10^6$) that the shift in the energy of the nucleus due to the e- loop does gravitate

Story n. III - The vacuum energy in QFT

ii. *Consistency of the theoretical context*

Argument of large radiative corrections to λ:

– if we DISREGARD GRAVITY ⇨ large vacuum energy IRRELEVANT!

$$S[\phi, g] = S_{SM}[\phi, g] + \frac{1}{16\pi G}\int (R[g] - 2\lambda)\sqrt{g}$$

– in perturbative QFT with GRAVITY on Minkowski space ⇨ (ϕ_0, η) not a solution of Einstein's equation with $\lambda \neq 0$!

$$\begin{cases} \phi = \phi_0 + \delta\phi \\ g = \eta + h \end{cases}$$

$$R_{\mu,\nu}[\eta] - \frac{1}{2} R\,\eta_{\mu,\nu}[\eta] + \lambda_{QFT}\,\eta_{\mu\nu} = 8\pi G\, T_{\mu,\nu}[\eta, \phi_0]$$

$$\Rightarrow \quad \lambda_{QFT}\,\eta_{\mu\nu} = 0$$

⇨ perturbation expansion not reliable, when it is around a field configuration which is not a solution of the equation of motion !

Need for QFT on curved spacetime ! Effect of λ visible where space is not flat!

Trusting flat-space QFT for the nature of a term of Einstein equations is a delicate and possibly misleading step!

Summary

- **The cosmological constant λ is a natural component of Einstein's equations**
  - *it was never felt as a "blunder" by the GR community*
  - *it received little attention only because its effect is difficult to measure*

- **There is no "coincidence problem"**
  - *if we consider equiprobability properly*
  - *if we do not postulate an unreasonably strong cosmological principle*

- **It is a mistake to identify λ with the vacuum energy density**
  - *we do not fully understand QFT renormalization, and QFT interaction with gravity in non-Minkowski spacetime (i.e. our Universe)*
  - *QFT troubles are a high-energy problem, whereas λ is a low-energy problem*

Conclusions

- Testing current theories and possible alternatives: good science !

- Claiming that *"Dark Energy is a profound mystery"*: nonsense
  - ⇨ "Dark Energy" is a name for the observed acceleration of the Universe

- The acceleration of the Universe is: *predicted* by current thories
  well described by these theories
  measured

  ⇨ No mistery in λ !

⇨ *Mysteries help getting funds, but a sober scientific explanation is preferable in Science.*

Thank you !